ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
CJSSR

CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
Vol.03 No.01 (2025)

MEMORY TYPE FLOATING CONTROL CHARTS FOR MONITORING PROCESS COEFFICIENT OF VARIATION

Sawaira Kainat[1], Saadia Tariq[1], Samia Bashir[1], Zulaikha Mashkoor[1], Junaid Talib[1] and Wardah Imtiaz[2]

[1]School of Statistics, Minhaj University Lahore, Lahore, Pakistan.
[2]MPhil Education, University of Education, Bank Road Campus, Lahore, Pakistan.

Kainatswaira49@gmail.com.com
Research scholar Minhaj University Lahore, Pakistan
Sadiasajad100@gmail.com
Associate Professor Minhaj University Lahore, Pakistan
Samiaahmad2011@gmail.com
Lecturer Minhaj University Lahore Pakistan
zulaikhamashkoor@gmail.com
Lecturer Minhaj University Lahore Pakistan
junaidtalib64@gmail.com
Lecturer Minhaj University Lahore Pakistan
wardahhijabian@gmail.com
Research scholar University of Education Bank Road campus Lahore Pakistan
Corresponding Author: *Saadia Tariq*

Abstract

This Paper aims to create a new floating exponentially weighted moving average control chart and monitor the coefficient of variation by using a variable sample size technique. It can be built using a log transformation, and Monte Carlo simulations with various shift sizes can be used to study it. Average run lengths of ARL 250, 370, and 500 are used with shift size at n = 5, 8, and 12 to suggest a new CV floating EWMA control chart. The findings show that for various values of n, the recently proposed CV floating EWMA control chart coefficient of variation outline performs well as the shift sizes rise. Comparing this new CV floating EWMA control chart to the current coefficient of variation control chart, it is evident that the new proposed CV floating EWMA control chart yields superior outcomes. A practical example of the die casting hot chamber process, which produces zinc alloy for the sanitary industry, is also provided.

Keywords: Coefficient of Variation (CV), Average run length (ARL), Control charts, EWMA, CUSUM, Progressive mean (PM)

1 Introduction

Quality is an important component of a company's success and has helped significantly to the growth of the manufacturing, industrial, and service sectors. Control chart is a graphical symbol of statistical process monitoring. It is used to check out that procedure is either in control or out of control. The middle one is surrounded by the upper control limit, lower control limit, and central line on control charts. The procedure is called out of control if any point falls outside of upper and lower limit. Average run length (ARL) is used to compare the performance of two or more charts.

Control chart was first introduced by Walter Shewhart in 1924 and later it was extended by W. Edward. Roberts first proposed the use of the exponentially weighted moving average (EWMA) control chart in 1959. EWMA charts, which use a reversible weight scheme to take into account both previous and current observations, as opposed to Shewhart type charts, which only take into account current observations, are the most straightforward method for identifying moderate and small shifts. Test statistics for EWMA, like

$$Z_i = \lambda X_i + (1-\lambda) Z_{i-1} \text{ For } i = 1, 2, 3, \ldots\ldots.. z_0 = \mu_0$$

λ is the range of weighting constants.

1.1 Floating Control charts

The majority of graphics are created by adjusting the specified sample variance. Costagliola (2005) introduced the S2 EWMA chart as a tool for tracking process dispersion. He achieved this by using a logarithmic three-parameter transformation to derive the normal approximation for sample variance. Three logarithmic conversions are given below by Costagliola (2005) :

$$T_c = a_T + b_T \ln(S_i^2 + C_T)$$

Here $S_i^2 = \frac{\sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2}{n-1}$, X_{ij} represent j[th] utilizing a mean-centered μ_0 normal distribution as a reference, make an observation from the i[th] n-fold random sample.

Constant are defined as

$$a_T = A_{T(n)} - 2B_{T(n)} \ln(\sigma_0),$$

$$b_T = B_{T(n)}, \; c_T = C_{T(n)} \sigma_0^2$$

With respect to the logarithmic transformation, the three parameters given in the equation make up the floating T-S^2 control chart that was first proposed.

The plotted statistic is as follows:

$$FT_i = \frac{\sum_{k=1}^{i} T_k}{i}$$

is based on a normal distribution with a mean $\mu_{T(n)}$ and variance $\text{var} = \frac{\sigma_T^2(n)}{i}$.

Control Limits are given as:

$$UCL = \mu_{T(n)} + k_T \frac{\sigma_{T(n)}}{\sqrt{i}} ,$$

$$CL = \mu_{T(n)}$$

$$LCL = \mu_{T(n)} - k_T \frac{\sigma_{T(n)}}{\sqrt{i}}$$

Range indicates by k_T .

1.2 Coefficient of Variation (CV)

The "coefficient of variation" (CV) is the ratio of the standard deviation to the mean. When the process standard deviations are a linear function of the mean and periodic fluctuations in the mean are anticipated, the process is referred to as the coefficient of variation in a control chart. For a sample, the coefficient of variation is:

$$CV = \frac{s}{\bar{x}} \times 100$$

CV for the population is:

$$CV = \frac{\sigma}{\mu} \times 100$$

Using the variable sample size (VSS) approach, Amdouni, Castagliola, Taleb, and Celano (2015) suggested a novel Shewhart control chart and determined that the process mean and standard deviation were proportional rather than constant. Formulas were used to calculate the reduced average run length. Fixed Sampling Rate Shewhart chart was utilized for the CV to examine each chart's efficacy in a short-term environment.

Haq and Khoo (2019) presented two responsive EWMA (AEWMA) charts for analyzing the irregular changes in the CV and multivariate CV (MCV) while sampling from univariate and multivariate normally distributed processes. These charts are referred to as the AEWMA CV and AEWMA MCV charts. To ascertain the run length properties of the proposed control charts, we performed extensive Monte Carlo simulations. The AEWMA CV chart consistently and meaningfully beats the previous ideal EWMA and CUSUM CV charts in recognizing moderate-to-large variations in the process CV. Furthermore, the AEWMA MCV chart consistently and significantly beat the traditional Shewhart MCV chart.

Perdikis, Psarakis, Castagliola, and Celano (2021) introduced a novel Phase II EWMA-type chart that solves the run-length features of the chart and is distribution-free, based on the Wilcoxon signed-rank statistic. The Markov chain method was used to determine the Average Run Length for both the in-control and out-of-control processes. The usefulness of the proposed chart was associated with several modern nonparametric methods. Additionally, a system for choosing the ideal chart design was created.

Abid, Mei, Nazir, Riaz, and Hussain (2021) created a new CUSUM chart using the continuously weighted statistic from a moving average chart. The proposed chart was related to other charts that already existed based on the non-identical run length requirement. The run-length outline correspondence, which was based on Monte Carlo simulations comparing the interpretation of the proposed chart to the current control charts, indicated that the suggested chart performed the best.

Alabi, Adegoke, Adebola, Oseni, and Abbasi (2022) examined the performance of two EWMA-based CV charts using one-sided exponential weighting. Alabi, Adegoke, Adebola, Oseni, and Abbasi (2022) used one-sided exponential weighting to assess the performance of two EWMA-based CV charts. To illustrate the efficacy of the charts at several run durations, the simulation outcomes were displayed. The ranked set sampling-based EWMA-CV charts with uphill and downhill rankings performed better than the SRS-based charts, according to the data.

2 Research Design and Methodology

The T-Statistics, which is shown below, was utilized by Castagliola et al. (2005).

$T_i = a + b \ln(\hat{\gamma}_i - c)$

where (n_i) and $\hat{\gamma}_0$ affect the three constants (a, b >0, and c) in such a way that T_i roughly maintains a normal distribution with (0, 1). The $\widehat{?}_i$ approach underlying the definition of T_i is based on a right-skewed distribution with a unimodal form.

Given that its distribution is possibly best described by a three-parameter model with a, b, and c that lie between

$T_i = a + b\ln(\widehat{?}_i - c)$ should roughly resemble a random variable with a normal distribution (0, 1). Its distribution may have been best described using a three-parameter system with a, b, and c lying between

(c, +∞). The parameter can be obtained with ease by fitting a three-parameter log normal distribution, since the center line for the chart monitoring T_i is at CL=0. The ?

distribution has three quantiles that are selected.

More precisely, by employing

$F_{\hat{\gamma}}^{-1}(\alpha/n, \gamma_0) =$

$$\begin{cases} \dfrac{\sqrt{n}}{F_t^{-1}\left(1-\alpha+A/_{n-1}, \frac{\sqrt{n}}{\gamma}\right)} & if \quad \alpha \in (A,1) \\ 0 & if \quad \alpha = A \\ \dfrac{\sqrt{n}}{F_t^{-1}\left(A-\alpha/_{n-1}, \frac{\sqrt{n}}{\gamma}\right)} & if \quad \alpha \in (0,A) \end{cases}$$

In case $x_r = F_{\gamma}^{-1}\left(r/_{n(i)}, \gamma_0\right)$,

$x_{0.5} = F_{\gamma}^{-1}\left(0.5/_{n(i)}, \gamma_0\right)$

And $x_{1-r} = F_{\hat{\gamma}}^{-1}\left(1-r/_{n(i)}, \gamma_0\right)$ the quantiles r, 0.5, which might represent the median, and 1-r, which would represent the $\hat{\gamma}$ distribution, for the order. According to previous discussion, the parameters a, b, and c are represented individually below:

$$b = \frac{F_N^{-1}(n)}{\ln\left(\frac{X_{0.5} - X_r}{X_{1-Y} - X_{0.5}}\right)}$$

$$a = b \ln\left(\frac{X_{0.5} - X_r}{1-\exp\left(\frac{F_N^{-1}(r)}{b}\right)}\right)$$

$$c = X_{0.5} - e^{-\frac{a}{b}}$$

For a normal distribution that lies between 0 and 1, the inverted distribution function is represented by $F_N^{-1}(r)$. There isn't another strict guideline to follow while implementing a value, and it's obvious that practitioners of left qualities can choose to leave.

3 Proposed Floating CV control chart

Examined Abbas's (2012) floating CC proposal, which was developed to account for process volatility. It is recommended that test statistics be generated using floating coefficient of variation control charts, or Floating CV CC for short in this study. The following are the test statistics that were created:

$$GT_i = \frac{\sum_{j=1}^{i} GY_i}{i}$$

The aforementioned data represents the mean of the three logarithmic conversions that were previously covered. The test data analysis To be consistent with the probability distribution, GT_i has a normal distribution.

$$\text{Mean} = E_{T_j}(\hat{\gamma}),$$

$$\text{Variance of } S_{T_j}(\hat{\gamma}) = \frac{G_{T_j}(\widehat{\gamma})}{i}$$

Control limits (CL), including upper control limit (UCL), center limit (CL), and lower control limit (LCL), can be expressed in the following ways due to this tangential statement:

$$UCL = E_{T_j}(\hat{\gamma}) + K\frac{G_{T_j}(\widehat{\gamma})}{i},$$

$$CL = E_{T_j}(\hat{\gamma}),$$

$$LCL = E_{T_j}(\hat{\gamma}) - K\frac{G_{T_j}(\widehat{\gamma})}{i}$$

In this instance, K can be used to represent the width of control limitations. The suggested scheme's ARLs with broader limits that are either larger or smaller cause the ARL_0 to continue adjusting by a constant amount. Equation provides the form of the newly assumed floating coefficient of variation:

$$UCL = E_{T_j}(\hat{\gamma}) + K\frac{G_{T_j}(\widehat{\gamma})}{i^{p+0.5}},$$

$$CL = E_{T_j}(\hat{\gamma}),$$

$$LCL = E_{T_j}(\hat{\gamma}) - K\frac{G_{T_j}(\widehat{\gamma})}{i^{p+0.5}}$$

ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
CJSSR

4 The Simulation-based Study

Since the suggested chart is based on a simulation study with a 1000×5 random sample with n=5 from a normal distribution, it is accepted as being in-control. With the smoothing parameter k = 4.8, the values ARL_0 = 250, a = 9.8864, b = 6.8112, and c = -0.1426 are obtained.

Table I: Comparison of Floating CV chart at different n and different size of shifts when ARL_0= 250

Shifts Size in CV	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
ARL n =5	250	25.687	11.5335	7.207	5.2868	4.2495	3.6161	3.1888	2.8734	2.6696	1.5249
ARL n =8	250	19.9726	8.991	5.8063	4.4378	3.6922	3.1922	2.8666	2.6122	2.4199	1.4615
ARL n =12	250	15.184	6.8123	4.6065	3.566	2.9915	2.6202	2.363	2.1815	2.0215	1.1697

As we raise the sample size from 5 to n = 8, the run-length value is 19.9726, as can be readily seen from Table 1's results when the shift size is at 10%. The similar value of average run length when n = 5 is 2,5.687. Furthermore, when we increase the sample size from 8 to 12, the equivalent result is 15.184. This means that the suggested chart works well at n=12, moderately well at n=8, poorly at n=5, and vice versaAs the shift size increases from 10% to higher, that is, 20%, 30%, up to 100%, the values of run-length at various sample sizes decrease as the shift size grows at its comparison when ARL_0 is 250. The graphical presentations in Figure 4.1 show minor differences in the fixed value of average run length at various n sizes.

ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
CJSSR

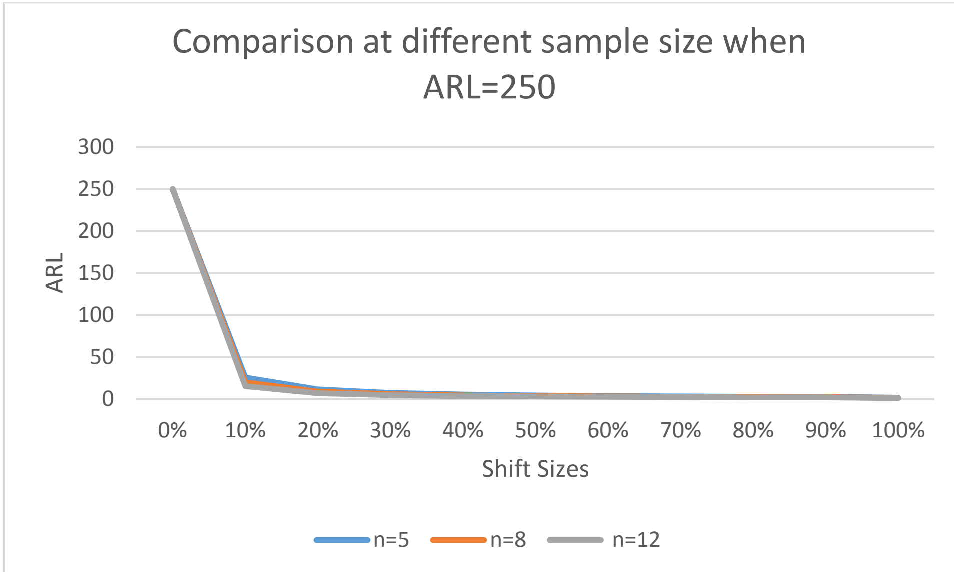

Figure 1: The performance of the floating CV control chart when the ARL is out of control at ARL = 250 for λ = 0.25 and n = 5, 8, and 12

Table 2: When ARL_0 =370, a comparison table of the floating coefficient of variation chart with varying n values at varying shift sizes is displayed.

Shift Size in CV	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100 %
ARL at n=5	370	31.657	13.600	8.615	6.449	5.219	4.442	3.860	3.470	3.221	1.938
ARL at n=8	370	22.390	9.6524	6.154	4.745	3.938	3.390	3.011	2.758	2.541	1.569
ARL at n=12	370	16.665	7.3335	4.8706	3.808	3.157	2.768	2.484	2.280	2.134	1.235

Considering the data in Table 2, it is evident that the average run length when n = 5 is definitely 31.6572 when the shift size is set to 10%. However, when n = 8 is used as a sample size, the run-length value increases to 22.37002. The suggested chart performs well at n=12, normal at n=8, minor at n=5, and vice versa. The corresponding figure, 16.6653, indicates even more the increase in sample size from 8 to n = 12. The run-length values decrease at different sample sizes when moving from 10% to 100% (i.e., 20%, 30%, and 100%) until they hit a comparison point at ARL_0 of 370. Variable numbers of shifts behave differently, as seen by the graphical displays in Figure 4.2.

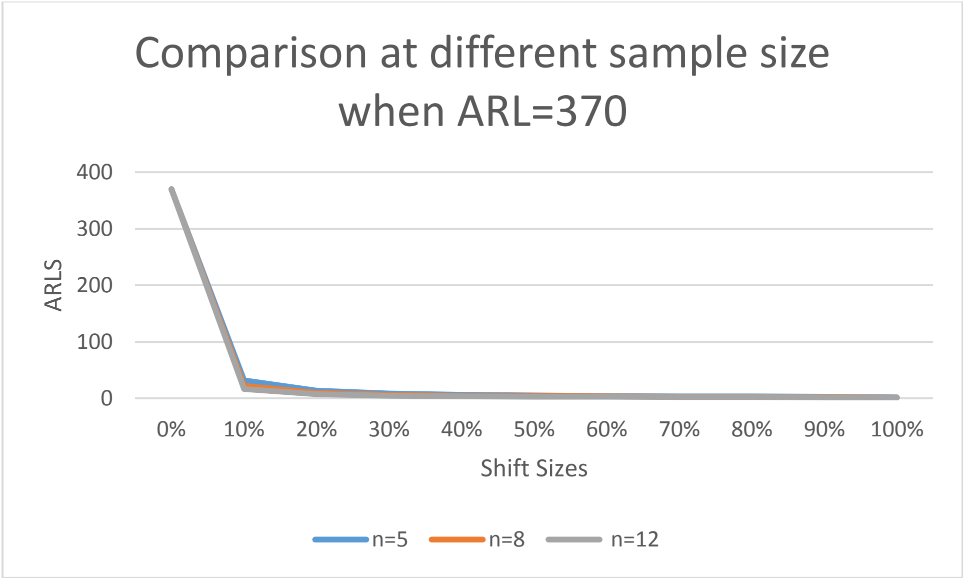

Figure 2: The performance of the floating CV control chart at ARL = 370 with λ = 0.25 and n = 5, 8, and 12 when the ARL is out of control.

Table 3: A chart representing the floating coefficient of variation with varying n values at varying shift sizes when $ARL_0 = 500$.

Shift Size in CV	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100 %
ARL at n=5	500	36.170 5	14.724 7	9.233 5	6.82 0	5.53 9	4.68 1	4.08 7	3.70 7	3.38 1	2.036
ARL at n=8	500	23.882 2	10.095 3	6.552 7	4.94 9	4.09 7	3.52 9	3.16 4	2.87 8	2.64 0	1.658
ARL at n=12	500	18.213 2	7.812	5.186 6	4.00 1	3.33 4	2.89 5	2.59 1	2.39 0	2.23 5	1.312

The data presented in Table 3 makes it evident that, at a shift size of 10 percent, the average run length for n = 5 is 36.1705. However, when we expand the sample size from n = 5 to n = 8, the run-length number increases to 23.8822. Additionally, when we increase the sample size from 8 to 12, the equivalent value is 18.2132, leading us to the conclusion that the suggested chart performs well at n=12, normal at n=8, minor at n=5, and vice versa. When we raise the shift sizes from 10% to 100% (that is, 20%, 30%, and 100%), the run-length values at various sample sizes exhibit a decline until ARL_0 equals 500. The behavior of different numbers of shifts is depicted graphically in Figure 3.

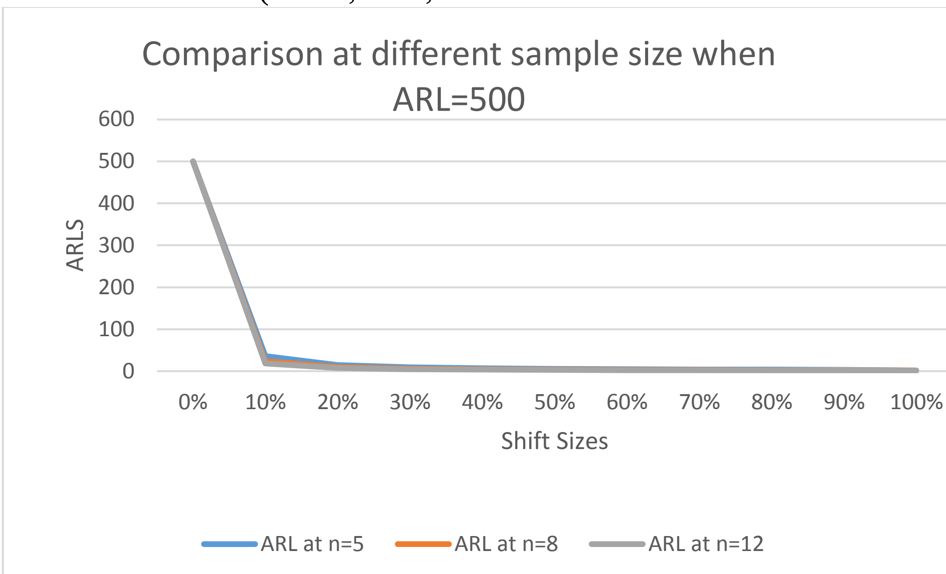

Figure 3: When λ = 0.25 and n = 5, 8, and 12 at ARL = 500, the floating CV control chart of the out-of-control ARL is compared.

4.2 Real Life Application

Using data from Castagliola et al. (2015) regarding the die casting hot chamber method used to create zinc alloy parts for the sanitary industry, we applied the proposed chart on an actual data set. Let X be the weight (in grams) of the remaining zinc material on the n identical component codes. Depending on what items are available and how well the lab can create combination parts, this weight could vary from inspection to inspection. For each of the parameter's a, b, and c that we generated from phase I data using SRS and RSS Castagliola et al. (2015), we have sample sizes of five. The process is believed to be under control. The normality of the RSS was established using the p-value of 0.65 obtained from the Anderson-Darling test, and the same procedure was applied to prove the normality of the T_t statistic. Phase II employed the projected values to track process CV at ARL_0 = 200. The plotting statistic (Et), the LCL, the UCL, and 100 samples of size 5 are all included in the phase II data set, which is displayed in Table 6. These values were all calculated using the RSS scheme. While the first fifteen numbers originated from the in-control process, the following values came from a shifted process with s = 2. The SEWMCV control chart, displayed in Figure 1, is out of control at observation 22.

The out-of-control signal on the 17th observation was discovered by the REWMCV control chart, which is depicted in Figure 2. It is concluded from these results that when log transformation is used, REWMCV control charts perform better than SEWMCV control charts.

To find the coefficient of variation (CV), the mean and variance of the given data are generated. The progressive mean of the normalized sample coefficient of variations was determined following the application of log transformation. A control chart is then created utilizing progressive mean statistics for coefficient of variation in order to track the effectiveness of progressive dispersion floating EWMA, as shown in Figures 4 and 5.

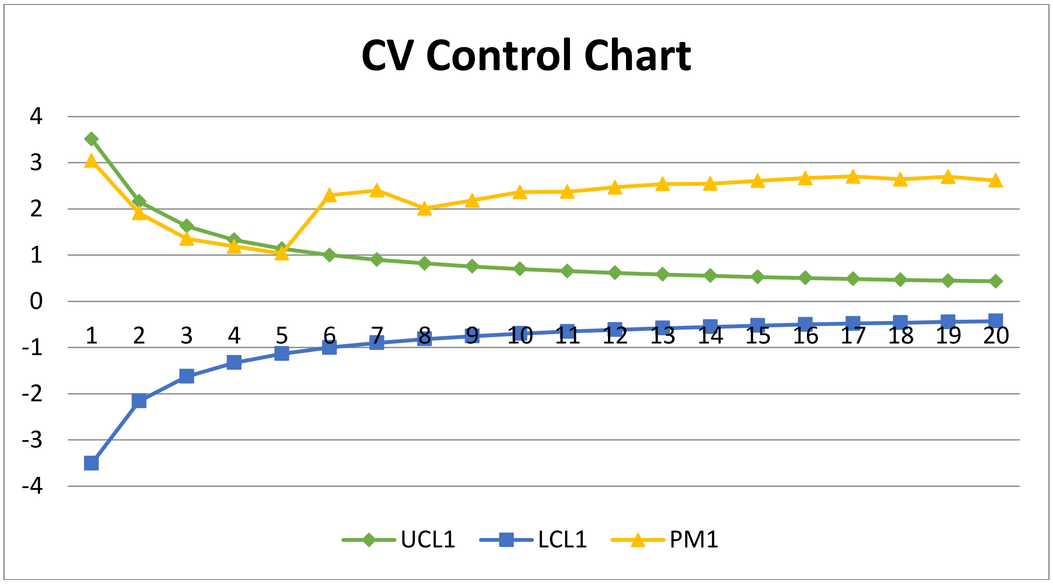

Figure 4: Chart Output for a Floating CV Control Chart's Process Dispersion

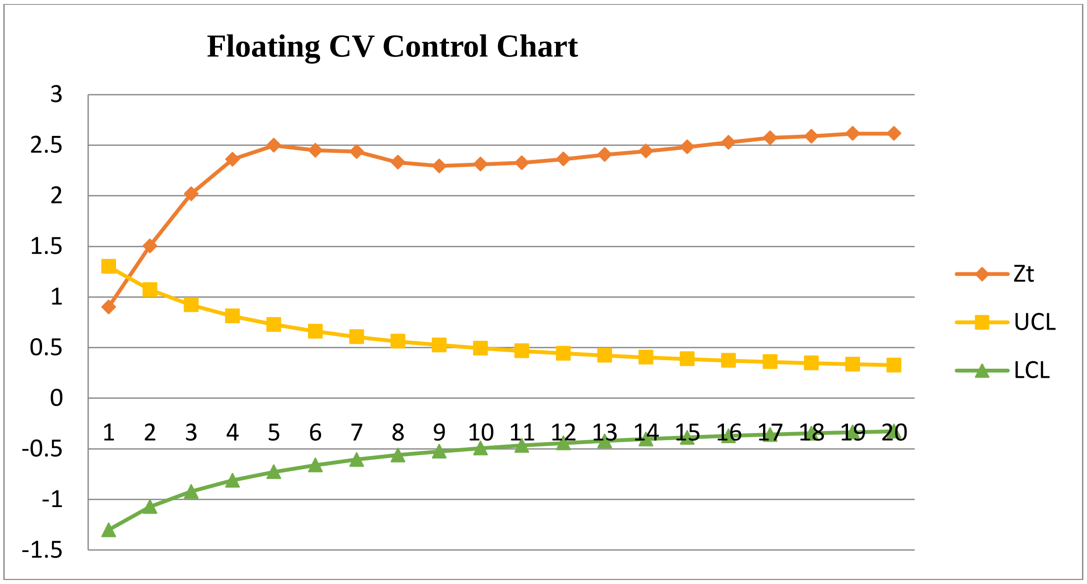

Figure 5: Floating CV Control Chart for Process Mean Output Chart

In contrast to our proposed floating coefficient of variation (CV) control chart, Figures 4 and 5 show that the coefficient of variation control chart produced a total of just seven out-of-control signals. The findings indicate that in terms of performance and the speed at which it can detect out-of-control signals, the recommended chart performs better than the CV control chart used in competition

Table 6: Comparing the existing control chart to the suggested floating CV control chart Comparison of the proposed control charts $ARL_0 = 370$ with the current control chart

	Floating CV Control Chart			CV Control Chart		
Shifts	n = 5	n = 8	n = 12	n = 5	n = 8	n =12
0%	370	370	370	370	370	370
25%	10.537	7.518	5.8352	12.52	8.17	6.56
50%	5.219	3.9389	3.1571	6.867	5.88	2.96

Table 6 provides clear proof that the suggested Floating CV control chart yields values for Average Run Length (ARL) of 10.537 at n=5, 7.518 at n=8, and 5.8325 at n=12, when a 25% shift size is used. On the other hand, with the same shift size, the

ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
CJSSR

standard CV control chart produces values of 12.52 at n = 5, 8.17 at n = 8, and 6.56 at n = 12. This is a highly significant number, indicating that the recommended plan outperforms the baseline system in terms of effectiveness. According to the suggested Floating CV control chart, the values at n=5, n=8, and n=12 are 5.219, 3.9389, and 3.1571, respectively, when shift size is 50%. On the other hand, the traditional CV control chart provides values for Figures 6.867 at n=5, 5.88 at n=8, and 2.96 at n=12. The graphical behavior in Figures 6, 7, and 8 is relevant to this task.

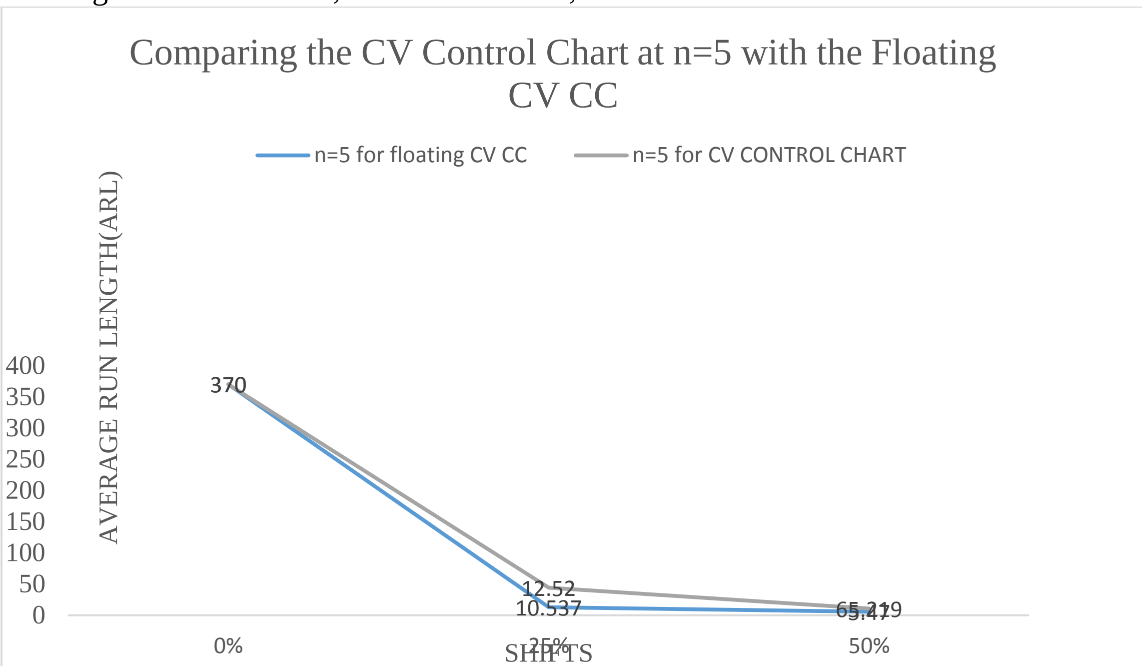

Figure 6: Chart output comparing the CV Control Chart at n=5 with the Floating CV CC

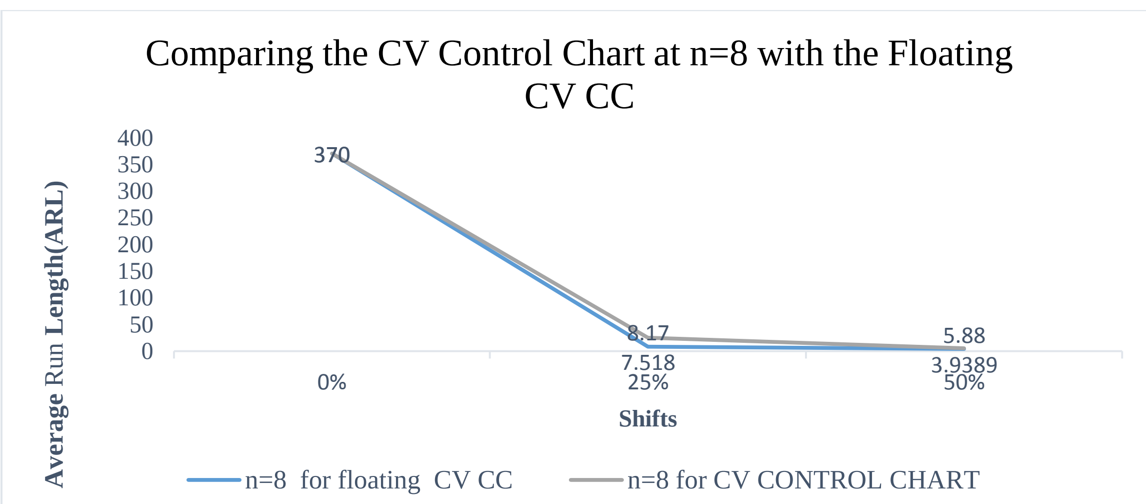

Figure 7: Chart output comparing the CV Control Chart at n=8 with the Floating CV CC

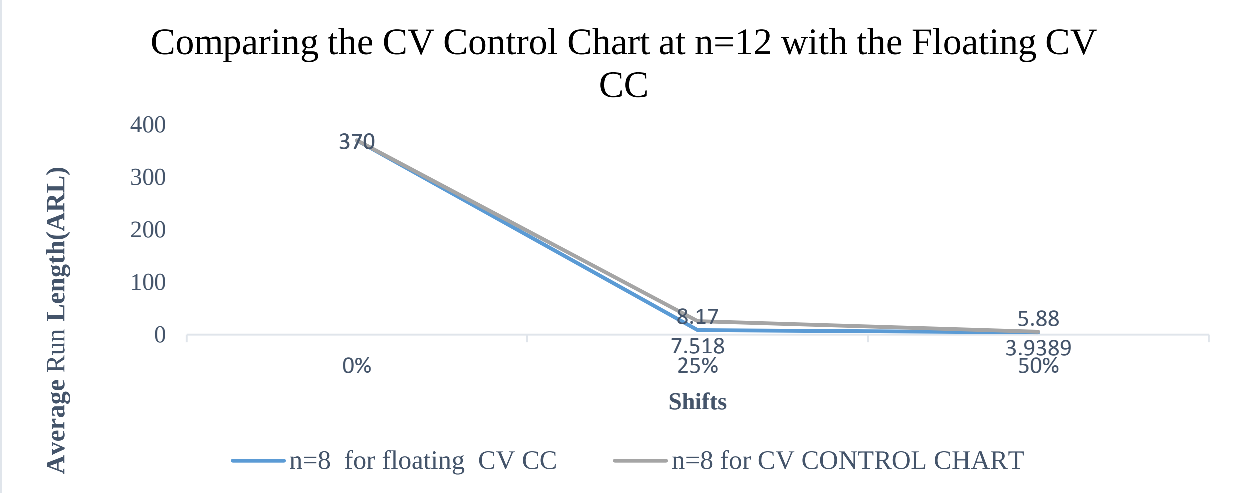

Figure 8: Chart output comparing the CV Control Chart at n=12 with the Floating CV CC12

Conclusion

The modified floating coefficient of variation control chart is shown in this study. Monte Carlo simulations with varying shift sizes are utilized to study it and build it utilizing a log transformation. The average run lengths of ARL 250, 370, and 500 are proposed as a floating coefficient control chart to monitor the behavior of process CV at n = 5, 8, and 12. The findings show that when shift sizes rise for various values of n, the suggested floating coefficient of variation chart functions well. This chart is contrasted to the competition's conventional coefficient of variation control chart to demonstrate that it yields superior outcomes. In this case, an actual example is also taken into account.

Data availability: All of the data generated or examined during this investigation is included in this published paper.

References

Amdouni, A., Castagliola, P., Taleb, H., and Celano, G. (2015). Monitoring the coefficient of variation using a variable sample size control chart in short production runs. *The International Journal of Advanced Manufacturing Technology, 81*(1),1-14.

Haq, A., & Khoo, M. B. (2019). New adaptive EWMA control charts for monitoring univariate and multivariate coefficient of variation. *Computers & Industrial Engineering*, *131*, 28-40.

Perdikis, T., Psarakis, S., Castagliola, P., & Celano, G. (2021). An EWMA-type chart based on signed ranks with exact run length properties. *Journal of Statistical Computation and Simulation*, *91*(4), 732-751.

Abid, M., Mei, S., Nazir, H. Z., Riaz, M., & Hussain, S. (2021). A mixed HWMA-CUSUM mean chart with an application to manufacturing process. Quality and Reliability Engineering International, 37(2), 618-631.

Alabi, O., Adegoke, N. A., Adebola, F. B., Oseni, B. M., & Abbasi, S. A. (2022). Exponentially weighted moving average control charts for monitoring coefficient of variation under ranked set-sampling schemes. *Journal of Statistical Computation and Simulation*, *92*(8), 1567-1589.

Castagliola et al. (2015). A New S^2-EWMA Control Chart for Monitoring the Process Variance. *Quality and Reliability Engraining International* 102(4), 1670-1682.