

ARTIFICIAL NEURAL NETWORK (ANN): PORTFOLIO OPTIMIZATION AND TRACKING ERROR

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Abstract

This study aims to investigate the application of the Artificial Neural Network (ANN) model in portfolio optimization, consider tracking errors in portfolio optimization, and evaluate their performance. The methodology comprises of two stages. At first, developing optimized portfolios based on market capitalization and risk using the ANN model. Second, optimizing portfolios while considering tracking error with the ANN model. It utilizes monthly prices data of listed companies on the Pakistan Stock Exchange and the KSE-100 index as a benchmark, ranging from December 31st, 1999, to December 31st, 2023. Performance Metrics like Mean Squared Error (MSE), Mean Absolute Error (MAE), and Root Mean Squared Error (RMSE) assess prediction accuracy. The findings of this study indicate consistently low Mean Squared Error (MSE) across all portfolios, affirming the efficacy of the ANN model in portfolio optimization and considering tracking error in portfolio optimization. Comparing the tracking error portfolios with market capitalization and risk portfolios reveals that the MSE values for tracking error portfolios are lower. This suggests that tracking error portfolios perform better. Researchers and financial analysts used traditional approaches to build portfolios to balance risk and return that depend on fixed assumptions. This research adds value to the existing literature on performance measures. Furthermore, this research uses the ANN model in portfolio optimization while considering tracking errors.

Keywords *Portfolio Optimization, Tracking Error, Artificial Neural Network (ANN), Mean Squared Error (MSE), Mean Absolute Error (MAE)*

1. Introduction

Investment is an important decision in an individual's life, whenever an individual has savings, he/she wants to invest his/her savings to increase his/her wealth or income (Fadi Alkaraan, 2013). The objective of investment varies from individual to individual and most of the investors or managers properly plan their investment according to risk tolerance, time horizon, and return which they want to get. There are three types of investors according to portfolio theory concerning risk: 1) risk-averse 2) risk-neutral and 3) risk lovers or takers. By nature, most investors are risk-averse and expect high returns. One of the assumptions of portfolio theory is that the investors maximize the utility of their portfolios by maximizing their returns but the difference in attitudes and behavior of investors towards the risk leads to the adoption of the different investments (Brown, 2002). The investment process involves five main steps. First, investors need to set their investment objectives; secondly, allocate the assets amongst the major allocated assets in the capital market. Thirdly, they choose a strategy for their portfolios that matches their

objectives. After that, they pick the specific investment to include in their portfolio. Finally, they evaluate how well their portfolio is doing compared to a benchmark they set.

In the dynamic landscape of financial markets, investors always have the challenge of creating investment portfolios that find the balance between risk and return. Moreover, risk and return play a very important role in making any investment decision. For instance, decisions such as “Should the investment be made?” and “Which securities should be included in the portfolio?” Furthermore, a portfolio is the collection of different financial securities such as shares, stocks, bonds, debt instruments, cash, and cash equivalents that an investor owns to achieve their financial goals. Portfolio management plays a very important role in investment decisions but there is a most complex problem which is portfolio optimization. The primary aim of portfolio optimization is to achieve the optimal balance between maximizing returns and minimizing risks. Several theories such as Portfolio Theory, and Modern Portfolio Theory (MPT) have been devised to address this objective and provide frameworks for effective portfolio allocation strategies (Rahmani, 2024).

Simply, portfolio theory is about risk and return preferences and opportunities to find the connection between maximizing return and minimizing risk. In portfolio theory, a portfolio represents the combination of investments that offers the highest potential return for the lowest possible risk (Mangram, 2013). However, the limitation of portfolio theory is that it focuses solely on risk and return preferences, without considering the relationships between different assets. There is no diversification¹ among assets so, in 1952, Harry Markowitz an American Economist, introduced a mathematical solution for the optimization problem to meet the investor needs of maximization of returns with minimum risk, which laid the foundation for modern portfolio theory (MPT) or mean-variance model. The Mean-Variance Model is a mathematical approach to constructing investment portfolios to achieve an optimal balance between risk and return. According to Markowitz, in Modern Portfolio Theory there are two types of risks: systematic risk and unsystematic risk. These are the two primary components that make up a security's overall risk in the context of a portfolio. MPT considers these two categories of risk to be common across all portfolios. Firstly, Systematic risk is basically a type of risk that affects several assets to various degrees at the macro level (Ross, 2002). Its factors include things like the overall economic situation, encompassing factors such as inflation, interest rates, unemployment rates, currency exchange rates, and Gross National Product. It is difficult to hedge systematic or market risk. Unsystematic risks, also called specific risks are unique to a specific investment or industry. It can be diversified up to some extent like they can be reduced by spreading investments or using protective strategies. These risks come from factors like how well a company is managed, how an asset performs, and how competitive an industry is (Tim Weingärtner, 2023).

As practitioners seek to implement MPT principles, they often face the challenge of ensuring that their portfolios align closely with the chosen benchmark or index. This introduces the concept of tracking error which measures how much their investment choices differ from the benchmark. A typical setup for active managers responsible for beating a benchmark is a constrained portfolio. This setup has an error in that the portfolio

¹ Diversification is as adding the number of securities in portfolio the risk goes down.

manager doesn't take the entire portfolio risk into account, which leads to severely inefficient portfolios unless extra constraints or other factors are placed in existence. So, a portfolio's tracking error provides a measure of how closely it follows its benchmark index. Additionally, tracking error, when applied in portfolio optimization, makes sure that the portfolio's performance matches the benchmark. Basically tracking error² is a financial performance measurement that compares the return fluctuations of an investment portfolio to the fluctuations of a given benchmark to find out how they differ. And, the return fluctuations are determined by standard deviation (Huang, 2021).

In the last 20 years, advanced technology like Artificial Intelligence has grown quickly and is now used in many areas, including finance. During this time, various studies have looked into how AI is used in finance. Artificial intelligence provides different models that help the investors in portfolio optimization and there are no chances of inefficient results. AI-based prediction models improve the portfolio selection process by accurately forecasting stock returns (Mondolo, 2023). The most common prediction tool is Artificial Neural Network which is an algorithm inspired by the activity of human brain cells, learning from data patterns to forecast future trends. Previous studies suggest that ANN outperforms other prediction methods like logistic regression, vector autoregressive, autoregressive, and Autoregressive integrated moving averages (Alev Dilek Aydin, 2015). ANNs are considered effective tools for predicting stock market behavior. However, their performance can fluctuate based on specific data they are applied to, making their accuracy vary across different situations. These models can be used to create dynamic and adaptive portfolio optimization strategies and can capture complicated relationships among variables. ANN technology does not have such limitations and provides a significant advantage in optimal portfolio selection.

Investors face difficulty choosing the best portfolio among different options. The Markowitz portfolio model helps by providing rules for optimization, allowing investors to find portfolios with the highest returns for a given level of risk or the lowest risk for a desired level of return. It creates an Efficient Frontier (EF) which contains all optimal portfolios. Typically portfolios are formed using historical time series data, but it may not accurately predict future conditions. So, the objective of this study is to apply the ANN model in portfolio optimization, in portfolio optimization by considering the tracking error and also to check the performance.

2. Literature Review

2.1. Traditional Approach

In 1952, Harry Markowitz wrote his doctoral dissertation in statistics, which formed the basis for Modern Portfolio Theory (MPT). The primary component of Markowitz's model was his clarification of how a portfolio's diversification is affected by the amount of securities it holds and their covariance relationships (Megginson, 1996). The results of his investigation, named "Portfolio Selection" (1952), were issued first time in *The Journal of Finance*. Later on, in 1959, he extended these findings. About three decades later, Markowitz received the Nobel Prize in economics and corporate finance in honor of his contributions to those fields through his work on MPT.

² For detail see "Tracking Error - Importance, How To Calculate and Limitations - Glossary by Tickertape"

The Capital Asset Pricing Model (CAPM), a further important advancement in capital markets theory, developed independently from Markowitz and Tobin's work. William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966) all independently formulated this idea. By giving investors a better framework for pricing securities based on systematic risk, the CAPM represents an important development in our knowledge of the equilibrium of the capital markets. When developing the CAPM, Sharpe's work from 1964 significantly enhanced the ideas of the Efficient Frontier (EF) and the idea of ³Capital Market Line (CML). Subsequently, Sharpe later received the Nobel Prize in Economics for his major achievements.

Lintner (1965) derived the CAPM from the perspective of a corporation issuing shares of stock, and a year later, Mossin (1966) also independently derived the CAPM, specifically incorporating quadratic utility functions. Since the pioneering works of Markowitz, followed by Sharpe (1964), Lintner (1965), and Mossin (1966), the field of Modern Portfolio Theory (MPT) has seen numerous expansions and iterations. The goal of this essay is to fill a perceived "simplicity" gap in the literature by highlighting an extensive failure amongst practitioners and theorists to take full advantage of the outstanding developments within the domain of technology and finance.

A study examines the enduring relevance of MPT, its insights into investor behavior, and its limitations, along with enhancements offered by Goal-Based Investing and Post-Modern Portfolio Theory (PMPT). The study reveals that while MPT helps investors choose optimal portfolios based on their goals and risk tolerance, it has significant limitations, such as its reliance on historical data and the exclusion of additional costs in risk-adjusted returns. Addressing these limitations through innovative approaches can lead to more effective portfolio selection and improved investment outcomes (Liu, 2023).

The emergence of modern portfolio theory (MPT) in 1952 revolutionized portfolio management by emphasizing the significance of diversification to mitigate risk, specifically through reducing specific risk. MPT introduced the idea of viewing assets as components of a portfolio rather than as individual investments, prioritizing low risk over high returns. This shift significantly impacted investors' behavior, influencing how they assess assets and perceive the financial market. However, despite its widespread adoption and recognition, MPT has notable shortcomings, such as its unrealistic assumptions and failure to account for macroeconomic factors and the potential evolution of companies.

2.2. *Artificial Neural Network (ANN) Model*

The neural network was first proposed in 1943 by neurophysiologist Warren McCulloch and aspiring mathematician Walter Pitts, who set out to use electrical circuits to represent the operations of neurons (Rosenblatt 1958).

ANNs are like computer versions of our brain's information processing system. The neural network (NN) employs a feed-forward back-propagation procedure, where a single unit, receives input signals from a function and converts them into output. These outputs are then passed on to subsequent units within the NN structure. The feed-forward process involves three components: input, hidden, and output. The input component receives external signals and is located in the innermost layer, while the hidden component, as the name suggests, remains concealed and does not interact with the external environment. The

³ CML shows relationship among risk and return of all portfolios.

output component transmits signals to the external environment and is located in the outermost layer (Williams, 1986). The back-propagation procedure is exceptionally valuable due to its ability to assign weights to multiple layers based on their relative significance in a consecutive manner (Rosenblatt, 1962).

A study identifies limitations with the single-layer perceptron, which leads to a stagnation in the field of ANN for approximately twenty years. In a significant development, Werbos (1988) introduces the backpropagation algorithm for multi-layer perceptrons, while Hopfield and Tank (1985) propose the energy model in ANNs. It is suitable for commercial use according to Rumerhart's (1986) multi-layer perceptron tests. The time required by the network calculation to arrive at a suitable approximation of an underlying problem is referred to as its complexity. An essential component of this computational complexity is algorithmic selection (Fonseca and Navarrese, 2002).

In a study by Callen, they used neural networks to try and predict how much money 296 companies on the NYSE would earn each month. They discovered that their neural network predictions had more noticeable errors compared to simpler linear prediction methods (Callen, 1996). Likewise, Thawornwong and Enke (2004) found that neural networks could predict future profits with less risk and better returns compared to basic strategies like just holding onto stocks, regular linear predictions, and a random guessing approach. However it's important to note that the real-world applications of these findings are somewhat restricted. Certain authors believe that the process of learning and using ANN is time-consuming, and they consider life to be too short for this extensive investment of time. Despite the limited number of studies in this area, most research indicates that ANN can effectively replace human decision-making in stock markets, offering superior outcomes. The capital structures of the top ten market capitalizations within the MSCI Emerging Index were investigated using ANN, support vector regression, and linear regression as forecasting techniques. Capital structure was defined by the ratio of total debt to total equity, in line with Tang et al. (1991). Various financial ratios, including profitability, liquidity, solvency, and turnover, were analyzed as factors influencing capital structure. Using logistic and hyperbolic tangent activation functions, the findings suggest that ANNs have significant potential to outperform traditional forecasting models, particularly with nonstationary data (Jesús Cuauhtémoc Téllez Gaytán, 2022).

2.3. *Portfolio Optimization using Artificial Neural Network (ANN)*

An ANN has been developed and utilized to forecast the price of stocks (Yetis et al., 2014). Majhi et al., (2007) examine the neural network's performance and contrast it with that of linear autoregressive and random walk models. According to Kumar et al. (2011), ANNs show a high endurance to uncertainty and can withstand noisy situations like stock market data. Because of these qualities, ANNs can be used to make well-informed choices regarding investments in stock markets.

Gokgoz and Sezgin-Alp (2014) launched another effort to assess the asset pricing models' forecasting ability through the use of artificial neural networks. Quah and Srinivasan (1999) use both basic and technical indicators to examine the behavior of stock market returns when ANNs are present. The effectiveness of these variables is investigated by combining the technical and basic variables both together and individually in this study. The study's main conclusion is that value stocks can be effectively separated from other stocks using (ANNs), and portfolios may be developed using this method. The paper presents a problem for the current portfolio creation processes (Jan, 2019).

In comparison to other stock prediction models, previous studies have shown that ANN, one of the machine learning models inspired by the activity of human brain cells, has more advantages in terms of speed, accuracy, and the amount of data that can be processed. The goal of diversification is to lower investment risk by distributing investment funds among many benchmark stocks. Choosing a portfolio is difficult because there are many stocks on the market. This study expands on several earlier studies that just used ANN or GA to make predictions without creating an ideal stock portfolio. The goal of this study is to forecast future stock values using ANN, and then create the best stock portfolios using GA to maximize return and minimize risk. According to the study's findings, using GA instead of the Single Index Model (SIM) technique produces better optimization indices (Mohammad Maholi Solin, 2019).

A new prediction-based portfolio optimization model was introduced that takes short-term investing into account. To forecast stock returns, the author used neural network predictors. From the prediction errors, which are based on a mean-variance model, the author developed a risk measure. The author tested the portfolio optimization model using actual data from the Brazilian stock market and included an evaluation of the normality of the prediction errors in his wide number of tests. The results show that non-normal stock return time series can produce Normal forecast errors. By incorporating short-term opportunities, the prediction-based portfolio optimization model also beat the market index and outperformed the mean-variance model. (Jorion, 2019) This literature review examined the use of ANN as a time series predictor for estimating returns in the context of the Pakistan Stock Exchange (PSX).

A study applied ANNs' projected returns in Markowitz's Mean Variance (MV) portfolio model to determine expected returns. Empirical data, including Pakistan Stock Exchange PSX listed stock closing prices, Karachi Inter Bank Offer Rates (KIBOR) as the risk-free rate, and the Karachi Stock Exchange KSE-all share index as a benchmark, was used for comparison. The portfolios' performance was evaluated under various constraints, such as budget, transaction costs, and turnover. Evaluation metrics like the Sharpe ratio and Information ratio were employed to measure portfolio value. The findings consistently highlighted the superior predictive capabilities and optimization potential of ANNs over the simple MV model, regardless of whether empirical or predicted data was utilized. Notably, the ANNs framework demonstrated exceptional performance in both long and short positions, resulting in significantly higher portfolio returns compared to the MV model. (Javed Iqbal, 2019).

The ability of ANNs and deep learning to analyze and comprehend complicated data, as well the scalability through parallel processing and optimization, are key components of the present State-of-the-art (Zappone et al., 2019). Effective optimization techniques are frequently needed to minimize or maximize objective functions in optimization situations. Even though ANNs are regarded as one of the greatest generic algorithms for problem-solving, they are still very much an unpredictable problem because they employ model weights and reorganize every iteration using the error method signal's backpropagation. By utilizing optimization approaches, ANNs can be improved and some of their drawbacks in selecting the optimal network structure could be eliminated (Abdolrasol et al., 2021).

2.4. *Portfolio Optimization with Tracking Error by using Artificial Neural Network (ANN)*

In recent years, deep learning has made a comeback and researchers are exploring its use in finance. Many studies focus on using deep learning to predict stock price movements, but not as much for directly optimizing investment portfolios. Index tracking is a popular passive investment strategy. To overcome this limitation, the author suggests a deep learning framework to optimize the index tracking portfolio.

Heaton et al. (2016) solved the index tracking problem as well as the improved index tracking problem by introducing the idea of a deep factor that takes into account the non-linear relationship of the input data using an auto encoder. The author used deep learning techniques like deep neural networks and deep auto encoders to monitor index performance. Additionally, they introduced a dynamic weight calculation approach to assess how individual stocks directly impact the index.

The empirical study examines the efficacy and viability of the index monitoring approach using historical data from the Hang Seng Index (HSI) and its components. The findings demonstrate that the deep neural network-based index tracking approach effectively tracks the index since it has a decreased tracking error (Hongbing, 2019).

Subject to a constraint that specifies a minimum expected excess return, the portfolio manager seeks to minimize the expected tracking error (Gnagi & Strub, 2020). According to Vochozka and Machová (2018), ANNs are one of the most widely used prediction techniques at the moment. ANNs are used by Dorneanu et al. (2011) to forecast business insolvency. The authors claim that because the proportion of prediction accuracy is better when using ANNs than when using traditional approaches, the usage of ANNs for prediction is very useful. Neural networks have been applied to a variety of fields, including the processing of signals, financial securities, and the detection of patterns.

3. Data and Methodology

The methodology of this study is divided into two parts by considering the objectives of the study. In the first stage, portfolios are created using market capitalization and risk assessment as the two main determinants. A more detailed explanation of these points is given below:

1. For market capitalization, companies are arranged in ascending order according to their market capitalization. The first hundred companies (50%) are classified as small size, while the subsequent hundred (50%) are considered big size. Twenty portfolios are then constructed in deciles, resulting in ten portfolios for small size companies and ten for big size.
2. Similarly, in the case of risk based portfolios, the risk is assessed using the Beta (risk) metric as shown in Equation 1. Companies are sorted from lowest to highest risk values, with the initial hundred (50%) considered low risk and the remaining hundred (50%) as high risk. Decile-based portfolios are then created, leading to ten portfolios classified as low risk and another ten as high risk.

$$Beta = \frac{Covariance(r_i r_m)}{Variance(r_m)} \dots\dots\dots (1)$$

Where r_i is the return on asset i , r_m is return on market which is taken as Karachi Stock Exchange KSE-100 index.

In the second stage, portfolios are created based on tracking error differences. Tracking errors can be expressed by a variety of statistical measures. For example, the correlation coefficient is a straightforward tracking measure. Other popular tracking measures include the first or second moments of the deviations between portfolio and benchmark returns. As a first tracking error measure, we use the square root of the non-central second moment of these deviations i.e.

$$TEV = \sqrt{\frac{\sum(R_{pk}-R_b)^2}{n-1}} \dots\dots\dots (2)$$

In this equation, R_{pk} represents tracking portfolio return in period k, R_{bk} shows the return of the benchmark portfolio set by an investor in period k, and “n” represents the sample size. Therefore, the KSE-100 index is used as the benchmark in this study. TE is a tracking error technique that is most frequently used in practice.⁴ This helps us understand how closely investment portfolios follow their intended benchmarks, which is important for managing risk and evaluating performance.

Artificial Neural Network (ANN) Model

ANNs consist of interconnected nodes called Artificial Neurons (AN) that are composed of three components i.e. input, hidden, and output layers explained below:

Input: The network takes in feature values from the dataset.

Activation Functions: Each neuron transforms the input using a function to capture a complex relationship. The Rectified Linear Unit, Sigmoid, Tangent Hyperbolic, and Softmax are the four most common types of activation functions. They are respectively defined in equation 3-6.

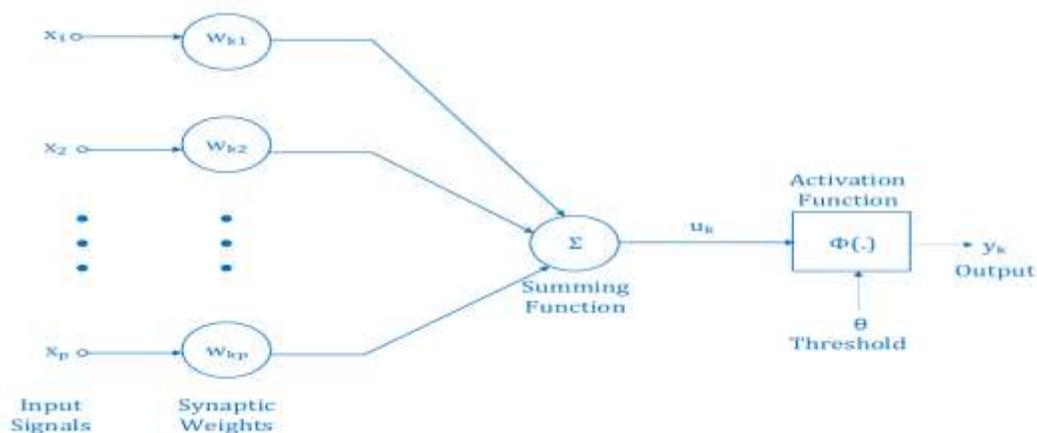
$$ReLU(k) = \max(0, k) \dots\dots\dots (3)$$

$$Sigmoid(k) = \frac{1}{1+e^{-k}} \dots\dots\dots (4)$$

$$\tanh(k) = \frac{e^k - e^{-k}}{e^k + e^{-k}} \dots\dots\dots (5)$$

$$Softmax(k) = \frac{e^{ki}}{\sum_j e^{kj}}, \forall i = 1, 2, \dots, n \dots\dots\dots (6)$$

Output: The network delivers the final computed predictions. The architecture of an ANN featuring a single hidden layer is depicted in Figure 1.



⁴For detail see “How to Calculate Tracking Error in Excel (with Detailed Steps) (exceldemy.com)”

Figure 1. Structure of Artificial Neural Network

A neural network consisting of interconnected links, each assigned weights as shown in Figure 1. A neuron can have a lot of input and only one output. There are weights and biases used in this model which are hyper parameters. The flow of the input signal (X_i) is multiplied by weights (W_i) and then the sum of all the inputs that have been weighted before. Then the weighted total ($W_1X_1 + W_2X_2 + \dots + W_kX_k$) is called the linear output (u_k) (Prashant Mahasagara1, 2017). The outcome of the sum is then processed by an activation function, represented by $f(u_k)$, which is usually composed of logical expressions. If it meets the threshold then it will give the output but if it doesn't meet the threshold then it will go back which is called backpropagation and the weights and biases will be changed.

4. Result and Discussion

Table I summarizes the portfolio with low market capitalization. Under the traditional method, portfolio 6 stands out by delivering the maximum return of 8.9% for a given risk level and minimum risk of 7.2% for a given return, making it the efficient choice. In contrast, the ANN evaluation identifies portfolio 9 as offering the highest return for a given risk, while portfolio 6 retains the lowest risk for a given return. Thus the traditional method selects one optimal portfolio, whereas the ANN approach highlights two optimal portfolios, P6 and P9.

I. Risk & Returns of Low Market Capitalization Portfolios

Portfolios	Traditional Approach		ANN Model	
	Return	Risk	Return	Risk
P1	-0.0006990	0.082955736	-0.022127415	0.061527
P2	0.015778396	0.081279894	-0.017536602	0.047965
P3	-0.087195871	0.082255433	-0.116679575	0.052772
P4	0.049846974	0.086690909	0.02178639	0.058630
P5	0.040175438	0.092617838	0.007167062	0.059609
P6	0.089005884	0.072048643	0.02137812	0.004421
P7	0.055045127	0.080631494	-0.003294562	0.022292
P8	0.037591188	0.07690944	0.005031977	0.044350
P9	0.058758212	0.093557173	0.027122048	0.061921
P10	0.06278094	0.083614187	0.020823205	0.041656

Table II indicates accuracy metrics while utilizing an ANN model for various low market capitalization portfolios. The average absolute difference between predicted and actual values is measured by MAE. More accuracy is indicated by lower MAE values. P1 has the lowest MAE of 0.10667, indicating that its predictions were somewhat accurate when compared to the actual values. The average squared difference between predicted and actual values is measured by MSE. In comparison with MAE, it gives larger errors greater weight. Once more, higher precision is shown by lower MSE values (Zain, 2011). Portfolio P1 has the lowest MSE of all the portfolios at 0.02143. The extent of errors is measured by

RMSE, which is the square root of the MSE. Similar to MAE and MSE, higher accuracy is indicated by lower RMSE values. P1 has the lowest RMSE of 0.14638.

II. Accuracy Measurement of Low Market Capitalization Portfolios

Portfolios	MAE	MSE	RMSE
P1	0.10667	0.02143	0.14638
P2	0.13973	0.03331	0.18252
P3	0.1416	0.02948	0.17171
P4	0.12722	0.02806	0.16751
P5	0.14607	0.03301	0.18168
P6	0.21301	0.06763	0.26005
P7	0.19884	0.05834	0.24154
P8	0.14719	0.03256	0.18044
P9	0.14375	0.03164	0.17787
P10	0.1629	0.04196	0.20484

Table III shows portfolios like P5 and P6 demonstrate relatively high returns of 12.78% and 10.11%, respectively, but also come with higher risks of 9.992% and 8.031%, respectively. Conversely, portfolios such as P5 and P7 have higher risk but also high returns. In the traditional approach, there are two efficient portfolios which are Portfolio 5 and Portfolio 10. In contrast, the ANN model presents a different perspective, with notable discrepancies in returns and risks compared to the traditional approach. The ANN model assigns negative returns to some portfolios (P3, P5, and P8), indicating potential losses over the specified period. So after applying the ANN model two efficient portfolios which are portfolio 4 and portfolio 5 are found. Notably, Portfolio 5 is efficient in both approaches.

III. Risk & Return of High Market Capitalization Portfolios

Portfolios	Traditional Approach		ANN Model	
	Return	Risk	Return	Risk
P1	0.07454	0.07914	0.03985	0.04444102

P2	0.06623	0.0758	0.03199	0.04155947
P3	0.0902	0.07821	-0.0039	-0.01587677
P4	0.09747	0.06776	0.07232	0.04261071
P5	0.1278	0.09992	-0.1075	-0.13541889
P6	0.10119	0.08031	0.02635	0.00546662
P7	0.09731	0.09339	0.05373	0.04980027
P8	0.09374	0.08724	-0.0287	-0.03519357
P9	0.07231	0.06974	0.04167	0.03909374
P10	0.09144	0.0642	0.04758	0.02034013

In Table IV the high market capitalization portfolios' accuracy metrics provide information on how well they predicted the future. The comparatively high accuracy of P1 and P4 can be observed from their lower Mean Absolute Error, Mean Squared Error, and Root Mean Squared Error figures, leading to more accurate predictions (Mihir, 2019). On the other hand, portfolios P6 and P7 show noticeably less accuracy, which is indicated by more errors as compared to others. This suggests that the prediction is less accurate than others. Some, like P1, P2, P6, and P7, continue to perform rather slowly, but others, like P5 and P10, exhibit fluctuations, especially when it comes to MSE and RMSE.

IV. Accuracy Measurements of High Market Capitalization Portfolios

Portfolios	MAE	MSE	RMSE
P1	0.14069	0.03469	0.18626
P2	0.15532	0.03424	0.18504
P3	0.26243	0.09408	0.30673
P4	0.12507	0.02514	0.15857
P5	0.47653	0.23534	0.48512

P6	0.22094	0.07484	0.27357
P7	0.16937	0.04359	0.20877
P8	0.33618	0.12243	0.3499
P9	0.13254	0.03064	0.17505
P10	0.16748	0.04386	0.20942

In Table V the traditional approach shows varying levels of return and risk across the portfolios, with Portfolio 3 exhibiting the highest return of 8.285% at a given level of risk which is 6.9% and Portfolio 1 having the highest risk of 14.128%. Portfolio 8 has the lowest risk which is 5.1% at a given level of return which is 5.4%. Conversely, the ANN model presents different outcomes, with Portfolio 3 still showing the highest return but with a lower risk of 1.94%, indicating potentially more efficient risk management. However, Portfolio 7 under the ANN model stands out with a significant negative return of -11.19% and a high risk of -10.60%.

V. Risk & Returns of Low Risk Portfolios

Portfolios	Traditional Approach		ANN Model	
	Return	Risk	Return	Risk
P1	0.01758	0.14128	-0.0086	0.1151
P2	0.0577	0.08308	-0.0476	-0.0223
P3	0.08285	0.06912	0.03308	0.0194
P4	0.08226	0.06226	-0.0251	-0.0451
P5	0.07278	0.05729	0.01867	0.0032
P6	0.0479	0.05795	0.01412	0.0242
P7	0.05419	0.06014	-0.1119	-0.1060
P8	0.05486	0.05178	-0.0053	-0.0084

P9	0.05898	0.05932	0.02295	0.0233
P10	0.03114	0.05931	-0.0049	0.0233

Table VI shows how well different low-risk portfolios perform. The MAE, MSE, and RMSE are all ways to gauge the difference between the expected and actual performance of each portfolio. Lower values for these metrics indicate better accuracy. Looking at the values provided, it is noted that portfolios P1, P6, P9, and P10 have generally lower errors compared to the others, meaning they tend to be more accurate in predicting their performance. So, investors might prefer portfolios with lower error values for more dependable performance estimates.

VI. Accuracy Measurements of Low Risk Portfolios

Portfolios	MAE	MSE	RMSE
P1	0.13158	0.02615	0.16172
P2	0.29841	0.10534	0.32456
P3	0.19117	0.04976	0.22308
P4	0.29028	0.10741	0.32773
P5	0.1977	0.05411	0.23261
P6	0.1438	0.03379	0.18381
P7	0.40274	0.16611	0.40757
P8	0.20467	0.06017	0.2453
P9	0.13391	0.03603	0.18981
P10	0.15738	0.036	0.18975

Table VII presents the performance comparison of high-risk portfolios using both traditional methods and an ANN model. In the traditional method, higher returns generally correspond to higher risks, with some exceptions and efficient portfolios are 1 and 10. However, the ANN model appears to offer a different risk-return profile, sometimes

generating lower risks for comparable returns compared to the traditional method. The ANN model shows portfolio 1 and Portfolio 8 as efficient portfolios it is found that portfolio 1 is efficient in both approaches from a perspective of risk.

VII. Risk & Returns of High Risk Portfolios

Portfolios	Traditional		ANN Model	
	Return	Risk	Return	Risk
P1	0.05595	0.06595	0.00508	0.01508
P2	0.0743	0.08285	0.0177	0.02624
P3	0.09783	0.085	0.02957	0.01674
P4	0.02638	0.09878	-0.0313	0.04110
P5	0.04557	0.10527	0.01242	0.07211
P6	0.07066	0.10972	0.03104	0.07009
P7	0.01818	0.12695	-0.0233	0.08545
P8	0.10919	0.14453	0.07466	0.10999
P9	0.11676	0.16652	0.05333	0.10310
P10	0.14064	0.22134	0.05827	0.13897

According to Table VIII, P5 is the most accurate in predicting the performance of high-risk portfolios because it has the lowest MAE, MSE, and RMSE of any portfolio. P10 has the least accuracy in predicting the performance of high-risk portfolios out of all the portfolios, as seen by its highest MAE, MSE, and RMSE.

VIII. Accuracy Measurements of High Risk Portfolios

Portfolios	MAE	MSE	RMSE
P1	0.19035	0.05087	0.22554

P2	0.20213	0.05661	0.23792
P3	0.21393	0.06825	0.26126
P4	0.19075	0.05768	0.24016
P5	0.13888	0.03315	0.18208
P6	0.15403	0.03963	0.19907
P7	0.15818	0.0415	0.20372
P8	0.14292	0.03453	0.18583
P9	0.20046	0.06342	0.25184
P10	0.24451	0.08238	0.28701

Table IX displays the outcomes of portfolio optimization using tracking error employing both traditional methods and an ANN model. The table compares the return and risk metrics for ten different portfolios (P1 to P10). In the traditional approach, portfolios demonstrate positive returns ranging from 0.00414 to 0.14226, with corresponding risk levels between 0.00454 and 0.22819. Conversely, the ANN model presents differing results, with returns ranging from -0.0217 to 0.12177 and risk levels spanning from -0.020592 to 0.20769. It is found that in traditional and ANN model portfolios 1 and 10 are efficient both have the same results.

IX. Risk and Returns of Portfolios Based on Tracking Error

Portfolios	Traditional Approach		ANN Model	
	Return	Risk	Return	Risk
P1	0.00414	0.00454	-0.02099	-0.020592
P2	0.00673	0.00804	-0.0217	-0.020403
P3	0.00923	0.00977	-0.0077	-0.007114
P4	0.0121	0.01338	-0.0096	-0.008371
P5	0.01555	0.01622	0.00038	0.001037

P6	0.0198	0.02495	0.00082	0.00596
P7	0.02563	0.02994	0.00151	0.00582
P8	0.03451	0.04628	0.01258	0.02436
P9	0.05103	0.07517	0.00646	0.03060
P10	0.14226	0.22819	0.12177	0.20769

Table X shows the accuracy measurements for ten different portfolios based on tracking error, assessed through MAE, MSE, and RMSE. All portfolios have low errors but across all the portfolios, P5 has the lowest MSE and RMSE which are 0.01518 and 0.1232 which indicates high accuracy of predictions. Similarly, portfolio 8 has the lowest MAE which is 0.08323 which indicates a high level of precision. Portfolio 9 has the highest MSE, MAE, and RMSE which are 0.4457, 0.15111, and 0.2111 showing low accuracy according to others.

X. Accuracy Measurements of Portfolios based on Tracking Error

Portfolios	MAE	MSE	RMSE
P1	0.12441	0.02513	0.15852
P2	0.14261	0.02845	0.16866
P3	0.09728	0.01689	0.12995
P4	0.10216	0.02175	0.14749
P5	0.09668	0.01518	0.1232
P6	0.09275	0.01899	0.1378
P7	0.10702	0.02412	0.1553
P8	0.08323	0.02193	0.14808
P9	0.15111	0.04457	0.21111
P10	0.09915	0.02050	0.14316

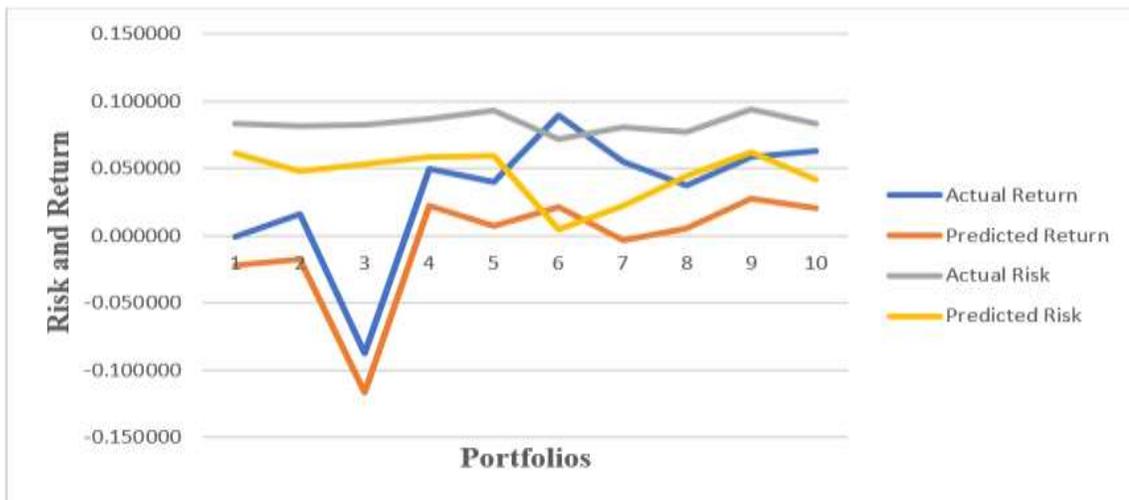


Figure 2. Comparison of Actual & Predicted Risk & Return of Low Market Capitalization Portfolios



Figure 3. Comparison of Actual & Predicted Risk & Return of High Market Capitalization Portfolios

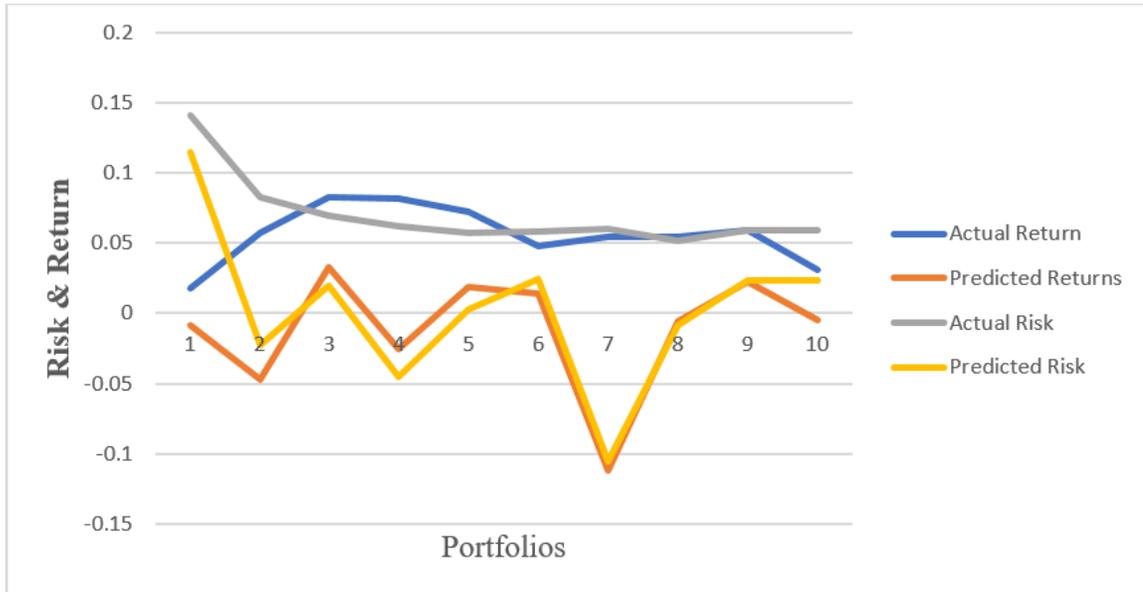


Figure 4. Comparison of Actual & Predicted Risk & Return of Low Risk Portfolio

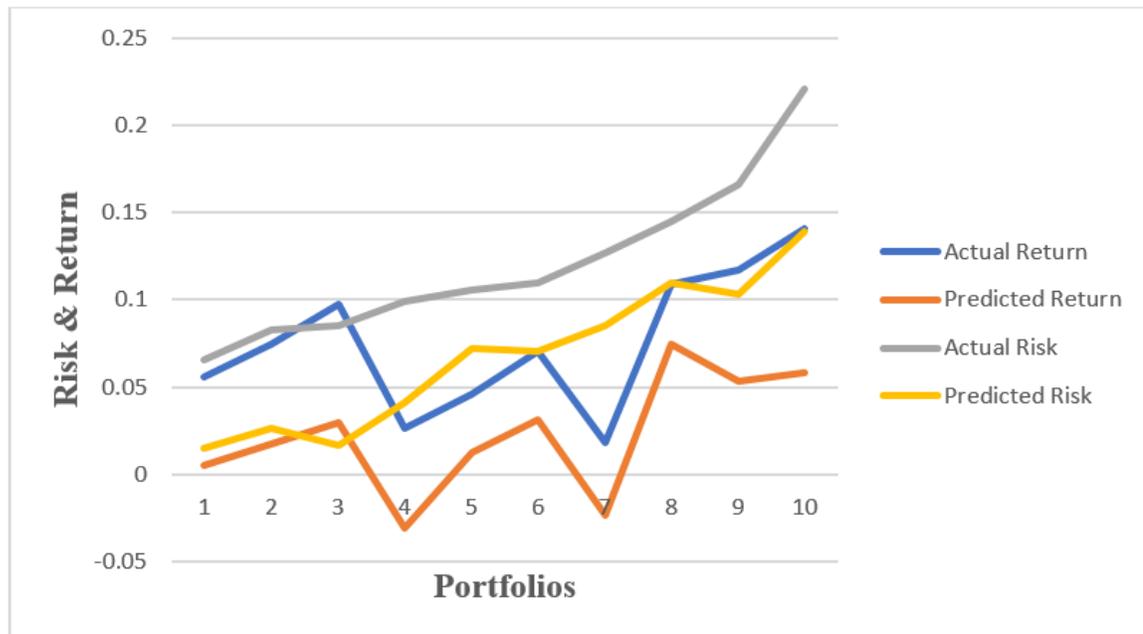


Figure 5. Comparison of Actual & Predicted Risk & Return of High Risk Portfolios

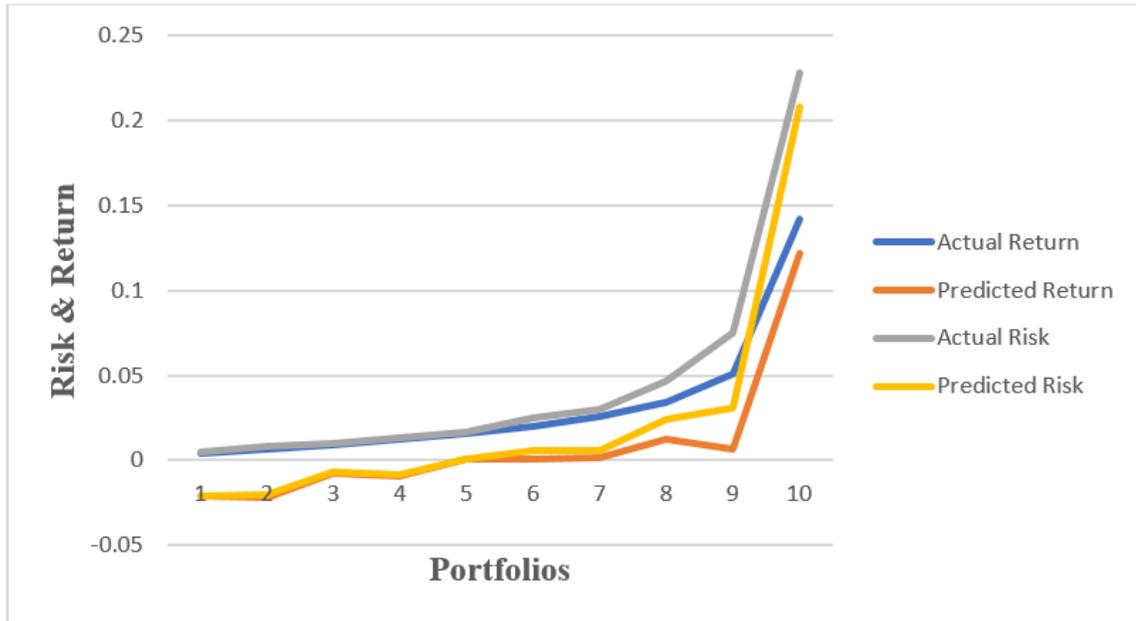


Figure 6. Comparison of Actual & Predicted Risk & Return of Portfolios based on Tracking Error

Figures 2 to 6 illustrates a comparative analysis between the Traditional approach and predictions generated by an ANN model regarding the risk and return profiles of portfolios: low market capitalization portfolios, high market capitalization portfolios, low risk portfolios, high risk portfolios, and portfolios based on tracking error. The x-axis shows the portfolios, while the y-axis represents the associated risk-return dynamics. The Mean Squared Error, Root Mean Squared Errors is notably low, indicating a high level of precision wherein the predicted values closely align with the actual data points. Furthermore, the depicted trend in these figures underscores a strong correlation between the accuracy levels of the predictions and their closeness to the actual values. This suggests a robust performance of the ANN model in capturing and predicting the risk-return relationship within the context of low market capitalization portfolios, high market capitalization portfolios, low risk portfolios, high risk portfolios, and portfolios based on tracking error.

XI. Comparison of Performance by Using Mean Squared Error

Portfolios	Low Market Capitalization Portfolios	High Market Capitalization Portfolios	Low Risk Portfolios	High Risk Portfolios	Portfolios based on Tracking Error
P1	0.021428371	0.034694109	0.025597665	0.05086904	0.025129431
P2	0.033314998	0.034240715	0.105336569	0.05660553	0.028446675
P3	0.029483704	0.09408425	0.049762694	0.06825469	0.016887704

P4	0.028060584	0.025144324	0.10740866 3	0.0576785 3	0.02175226 8
P5	0.033008375	0.235336768	0.05410738 6	0.0331539 9	0.01517826 6
P6	0.067627764	0.074839796	0.03378786	0.0396271 8	0.01898874 9
P7	0.058339689	0.043585912	0.16611228 5	0.0415024 4	0.02411806 3
P8	0.032559211	0.122429459	0.06017219 6	0.0345340 4	0.02192904 6
P9	0.031636164	0.030643421	0.03602615 8	0.0634243 8	0.04456942
P10	0.041957736	0.043856014	0.03600406 1	0.0823772	0.02050000

Table XI compares the performance of tracking error portfolios against low and high market capitalization portfolios and low and high risk portfolios based on Mean Squared Error. Lower MSE values indicate better performance due to smaller prediction errors. The MSE range for tracking error portfolios is 0.01517 to 0.04456, which is lower compared to the other portfolios. This indicates that the ANN model performs better in tracking error portfolios.

5. Conclusion

In the era of globalization, portfolio management plays a pivotal role in investment decisions, which includes putting financial securities together. Traditional portfolio optimization theories like MPT, CAPM, and Linter heavily rely on historical data, often leading to conservative results (Perold, 2004). Markowitz's method had two main problems: it didn't measure risk in the best way, and it wasn't great for long-term planning. Plus, it wasn't perfect for real-world situations, there are extra costs and complexities involved in making investment decisions (Kandahari, 2019). Previously researchers and financial analysts used traditional approaches like mean-variance analysis to build portfolios to balance risk and return which often overlook market dynamism because they depend on fixed assumptions. These theories overlook the potential predictive power of auxiliary data available during decision-making (Yang, 2023). Consequently, the emerging field of AI in finance although extensively explored in various domains includes the stock market, trading models, volatility forecasting, cryptocurrencies, derivatives, and credit risk in banks. However, there is limited research on AI's use in portfolio optimization and tracking error management. ANN, a form of soft computing, is highly accurate and extensively employed for predicting across various domains such as social sciences, engineering, economics, finance, and stock markets (Charles Korede, 2014). Previous research achieved high accuracy in stock price prediction using ANNs but did not advance further (Adebisi, 2014). Another study (Nelson, 1999), it was found that ANN models are generally accurate. They suggest that by making a few more modifications, these models could be improved further. So, this study aims to fill these gaps and extend the prediction to portfolio optimization and also to consider the tracking error to evaluate the performance of these optimized portfolios by using ANN.

In this study, Portfolios are constructed based on two criteria: Market Capitalization (High, Low) and Risk (High, Low). ANNs are employed to analyze these portfolios, using supervised learning techniques with returns of the companies as input and optimized portfolios as output. There were two hidden layers each containing 64 neurons and used sigmoid activation function because of the regression problem. After constructing the ANN model and utilizing it to predict optimal portfolios for each dataset, the subsequent step involves forming various alternative portfolios through a combination derived from the ANN output. So, the optimized portfolios of two categories using ANN in terms of risk and return are formed. Performance measurements such as MSE, MAE, and RMSE are statistical measures used to evaluate the accuracy of predictions made by a model. The results show low MAE, MSE, and RMSE across all portfolios, indicating accurate predictions. Comparing the tracking error portfolios with low and high market capitalization portfolios and low and high risk portfolios reveals that the MSE values for tracking error portfolios are lower. This suggests that tracking error portfolios perform better.

The empirical findings of this study underscore the significant advantages of employing ANNs for portfolio optimization, specifically in the context of the Pakistani stock market. The ANN model demonstrated high predictive accuracy, as evidenced by low MSE, MAE, and RMSE across all portfolios. This suggests that ANNs can effectively optimize portfolios based on criteria such as market capitalization and risk levels. Overall, incorporating ANNs into portfolio management offers a significant opportunity for both academic research and practical applications in finance. This integration supports the development of more intelligent and adaptive investment strategies suited to the complexities of modern financial markets. There are certain limitations found in this study. Portfolios are made on just market capitalization and risk basis, there are other factors on which basis portfolios can be generated like investment, profitability, and momentum which will help in future research. Secondly, optimization with tracking error is done in this study other factors or multiple constraints like risk tolerance, transaction cost, and liquidity requirements may also be used for future research to achieve a well-balanced and diversified investment strategy. Finally, only the non-financial sector is considered, this study might be repeated by using the data from the financial sectors.

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