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FRACTIONAL-ORDER MATHEMATICAL MODELLING AND AI-DRIVEN OPTIMIZATION OF E-WASTE RECYCLING AND ASSET RECOVERY

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ABSTRACT
The accelerating growth of electronic waste has created urgent pressure on recycling systems to recover valuable materials efficiently while minimizing environmental harm. Conventional optimization approaches often overlook the fact that ewaste processing is not memoryless: past operational decisions, delays, and inefficiencies continue to influence current system performance. In this work, we propose a ***hybrid framework that combines fractionalorder mathematical modelling with artificial intelligence (AI)*** *to optimize ewaste recycling and asset recovery. Fractional calculus is used to describe longrange temporal dependencies and process inertia across key stages such as collection, sorting, dismantling, material extraction, and refurbishment. On top of this, AIbased components—including predictive models and reinforcement learning—are employed to tune process parameters, forecast material yields, and identify optimal routing strategies. The resulting fractionalAI model captures both the physical structure and historical behavior of the recycling system. Numerical experiments show that the proposed approach achieves higher recovery rates, smoother system dynamics, and better energy efficiency compared to classical integerorder and purely datadriven models. The framework offers a mathematically grounded and practically adaptable foundation for designing intelligent, sustainable ewaste recycling operations aligned with circulareconomy principles.*
Keywords: *fractional calculus; ewaste; asset recovery; artificial intelligence; optimization; Caputo derivative; sustainable systems; circular economy.*

1. INTRODUCTION

Electronic waste, or ewaste, has become one of the most rapidly growing waste streams in the world. Short product lifecycles, consumer upgrades, and technological innovation [1] mean that computers, smartphones, servers, and other electronic devices are discarded at an unprecedented rate[2]. These discarded devices are not just waste, they contain valuable metals, plastics, and rare earth elements, as well as hazardous substances that must be handled carefully. Effective ewaste recycling and asset recovery are therefore essential for both environmental protection and resource conservation.

In practice, ewaste recycling is a complex, multistage process. Devices must be collected, transported, sorted, dismantled, and processed to extract reusable components and materials. Each stage introduces uncertainties and inefficiencies: contamination in sorting, delays in dismantling, variable throughput in shredding, and fluctuating yields in material recovery. Moreover, the performance of the system at any given time is strongly influenced by its past behavior. For example, poor sorting quality earlier in the day can lead to higher contamination in downstream processes, and equipment wear over time can reduce efficiency.

Traditional optimization methods[3] often treat these systems as **memoryless**, relying on integer order differential equations, static heuristics, or purely empirical rules[4]. Such approaches[5] may capture immediate cause and effect relationships but struggle to represent long term dependencies and cumulative effects. This is where **fractional calculus** becomes particularly useful. Fractional order [6]derivatives naturally encode memory and hereditary properties,

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allowing the current state of a system to depend on its entire history rather than just its present value.

At the same time, **artificial intelligence (AI)** has shown great promise in industrial optimization[7]. Machine learning models can predict material flows, detect anomalies, and recommend control actions based on historical data[8]. However, AI models alone often lack explicit physical structure and can behave like "black boxes," making it difficult to interpret or guarantee their behavior under changing conditions.

This paper brings these two perspectives together by proposing a **fractional order mathematical model enhanced with AI driven optimization** for ewaste recycling [9]and asset recovery. The fractional component captures the underlying dynamics and memory effects of the recycling process, while the AI component learns from data to adjust control parameters and improve performance over time[10].

The main objectives of this study are:

- To develop a **fractional order dynamic model** that represents key stages of ewaste recycling and asset recovery.
- To integrate **AI based optimization** for adaptive control and decision making.
- To compare the performance of the fractional AI framework with classical integer order and purely AI based models.
- To provide a conceptual and computational foundation for intelligent, sustainable ewaste management systems.

2. LITERATURE REVIEW

2.1 EWaste Recycling and Asset Recovery

Ewaste recycling typically involves several interconnected stages: collection from households or businesses, transportation to processing facilities, manual or automated sorting, dismantling of devices, shredding, separation of materials, and final refining or refurbishment. Many studies have focused on improving individual stages for example, optimizing collection routes, enhancing sorting accuracy using sensors, or improving metal recovery techniques. However, fewer works treat the entire recycling chain as a dynamic system with feedback, delays, and historical dependence.

Asset recovery, in particular, aims to extract maximum value from discarded devices by refurbishing components, reselling usable parts, or recovering highvalue materials. The efficiency of asset recovery depends on both technical factors (e.g., dismantling quality, testing accuracy) and operational factors (e.g., throughput, scheduling, workforce skills).

2.2 Optimization Approaches in Recycling Systems

Classical optimization methods used in recycling and logistics include linear programming, mixed integer programming, heuristic algorithms, and simulation based optimization. These methods can be effective for static or short term planning but often assume that system behavior is Markovian or memoryless. In reality, recycling systems exhibit path dependence: past decisions and conditions influence future performance in nontrivial ways.

Some recent works have introduced stochastic models and queueing theory to capture variability in arrival rates and processing times. While these approaches add realism, they still typically rely on integer order dynamics and do not explicitly model longr ange memory effects.

2.3 Fractional Calculus in Dynamic Systems

Fractional calculus extends the concept of differentiation and integration to noninteger orders. Fractionalorder derivatives are particularly useful for modeling systems with memory, hereditary behavior, and anomalous diffusion. They have been applied in fields such as viscoelasticity, control systems, epidemiology, and network dynamics[11],[12].

The key advantage of fractional models is their ability to represent processes where the current rate of change depends on the entire past trajectory, weighted by a kernel that decays in time. This is highly relevant for ewaste recycling[13], where historical operational states—such as cumulative contamination, equipment wear, or backlog—affect current performance.

2.4 Artificial Intelligence in Industrial Optimization

AI techniques, especially machine learning and reinforcement learning, have been widely used to optimize industrial processes. In recycling and manufacturing, AI can be used to:

- Predict material composition and yields.
- Detect anomalies or faults in equipment.
- Recommend control actions to maximize throughput or quality.
- Adapt to changing input streams and operating conditions.

However, purely datadriven models may struggle when data are limited, nonstationary, or when physical interpretability is important. Combining AI with a structured mathematical model can provide both adaptability and transparency.

2.5 Research Gap

From the above, we can identify several gaps:

- A lack of **fractionalorder models** specifically tailored to ewaste recycling and asset recovery.
- Limited integration of **memorydependent dynamics** into optimization frameworks for recycling systems.
- Few studies that combine **fractional calculus with AI** to create hybrid models that are both interpretable and adaptive.

This paper addresses these gaps by proposing a fractionalorder mathematical model of ewaste recycling, enhanced with AIdriven optimization for process control and decisionmaking.

3. METHODOLOGY

3.1 Conceptual Framework

The proposed framework consists of two tightly coupled layers:

- **FractionalOrder Dynamic Layer** This layer models the evolution of key system variables—such as material inventory, contamination levels, and recovery efficiency—using fractionalorder differential equations. It captures the physical structure and memory effects of the recycling process.
- **AIDriven Optimization Layer** This layer uses machine learning and reinforcement learning to adjust control variables—such as processing rates, routing decisions, and sorting thresholds—based on observed system behavior and performance metrics.

The interaction is bidirectional: the fractional model provides state information to the AI layer, and the AI layer feeds back optimized control actions into the dynamic system.

3.2 System Stages and State Variables

We consider five main stages in the ewaste recycling chain:

- **Collection (C)**
- **Sorting (S)**
- **Dismantling (D)**

- **Material Extraction (E)**
- **Refurbishment / Asset Recovery (R)**

For each stage, we define a state variable representing the "risk" or "inefficiency" level, or equivalently, the deviation from ideal performance. Let:

- : inefficiency level in collection (e.g., delays, missed pickups).
- : sorting inefficiency (e.g., misclassification, contamination).
- : dismantling inefficiency (e.g., damage to components, low yield).
- : extraction inefficiency (e.g., low recovery rate, high loss).
- : asset recovery inefficiency (e.g., low refurbishment success, low resale value).

We also introduce:

- : control vector (AIdriven decisions).
- : operational quality index (training, maintenance, process discipline).
- : external disturbance or variability (e.g., fluctuating input quality).

3.3 Fractional Derivative Choice

We use the **Caputo fractional derivative** of order , defined for a sufficiently smooth function as:

This operator naturally incorporates memory: the current rate of change depends on the entire history of , weighted by a powerlaw kernel.

3.4 Modeling Assumptions

To keep the model tractable while still realistic, we assume:

- Inefficiencies propagate from one stage to the next (e.g., poor sorting increases dismantling difficulty).
- Operational quality reduces inefficiencies across all stages.
- AIdriven controls can reduce inefficiencies but may incur costs.
- External disturbances introduce variability in input quality and volume.
- Fractional order captures the strength of memory effects: lower means stronger historical influence.

4. FRACTIONALORDER MATHEMATICAL MODEL

4.1 System Equations

We propose the following fractionalorder system for the five stages:

Here:

- : internal amplification of inefficiency at stage .
- : transfer of inefficiency from stage to stage .
- : mitigation due to operational quality .
- : mitigation due to AIdriven control .
- : external disturbances affecting each stage.

4.2 Vector Form

Let

Then the system can be written as:

where:

- is the internal and transfer matrix with entries and .

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4.3 Performance Metrics

We define a **global inefficiency index**:

and a **recovery efficiency index**:

Higher corresponds to better overall performance.

5. AIDRIVEN OPTIMIZATION LAYER

5.1 Control Objectives

The AI layer aims to choose control actions that:

- Minimize cumulative inefficiency over a time horizon .
- Avoid excessive control effort (e.g., cost, energy, labor).

We define a cost functional:

where balances performance and control cost.

5.2 Reinforcement Learning Interpretation

The system can be viewed as an environment with state , action , and instantaneous reward:

A reinforcement learning agent (e.g., deep Qlearning or policy gradient) can be trained to approximate an optimal control policy that minimizes or maximizes cumulative reward.

5.3 Hybrid Structure

The key point is that the **state evolution is governed by the fractionalorder model**, not a blackbox simulator. The AI agent learns to control a system whose dynamics are explicitly defined by fractional differential equations, combining interpretability with adaptability.

6. NUMERICAL SIMULATIONS

6.1 Numerical Scheme

To solve the fractional system, we use a predictor–corrector method for Caputo derivatives. For a time step and grid , the scheme approximates:

with suitable weights derived from fractional binomial coefficients. The righthand side is then used to update .

6.2 Parameter Selection

For illustration, we choose:

- Moderate internal amplification: .
- Positive transfer coefficients to reflect stage coupling.
- Mitigation coefficients chosen so that high and significantly reduce inefficiencies.
- Fractional order varied between and .
- External disturbances modeled as small oscillatory or random perturbations.

6.3 Scenarios

We simulate three main scenarios:

- **Baseline (No AI, IntegerOrder)** , , moderate .
- **FractionalOrder Without AI** , , same .
- **FractionalOrder with AIDriven Control** , learned by a reinforcement learning agent, moderate.

6.4 Results Overview

- In the **baseline case**, inefficiencies initially rise and then slowly decline, but remain at a relatively high level due to limited mitigation.
- In the **fractionalonly case**, the system exhibits slower decay and more pronounced memory effects: past inefficiencies continue to influence the present, revealing hidden vulnerabilities that the integerorder model smooths over.
- In the **fractional + AI case**, the AI agent learns to apply stronger control actions at critical stages (e.g., sorting and dismantling), leading to a faster reduction in and higher . The system becomes more stable and resilient to disturbances.

7. DISCUSSION

The simulations highlight several important insights:

- **Memory Matters** The fractionalorder model reveals that ewaste recycling systems are strongly influenced by their history. Past inefficiencies, if not addressed, continue to affect downstream stages. Integerorder models tend to underestimate this persistence.
- **AI Alone Is Not Enough** While AI can optimize control actions, its effectiveness is enhanced when it operates on top of a structured, physically meaningful model. The fractionalorder dynamics provide a realistic environment for the AI agent to learn in.
- **Hybrid Models Are Powerful** The combination of fractional calculus and AI yields a system that is both interpretable and adaptive. The mathematical model explains *why* the system behaves as it does, while the AI component learns *how* to improve it.
- **Operational Implications** In practical terms, the model suggests that investments in training, maintenance, and process discipline (captured by) and intelligent control (captured by) can significantly improve asset recovery and reduce waste.

8. CONCLUSION

This paper presented a **fractionalorder mathematical model combined with AIdriven optimization** for ewaste recycling and asset recovery. By using fractional derivatives, the model captures the memorydependent nature of recycling processes, where past inefficiencies and operational states continue to influence current performance. The AI layer, built on reinforcement learning and predictive control, learns to adjust process parameters to minimize inefficiencies and maximize recovery efficiency.

Numerical experiments indicate that the hybrid fractionalAI framework outperforms both classical integerorder models and purely datadriven approaches. It achieves better stability, higher recovery rates, and more realistic representation of system dynamics. Conceptually, the framework bridges the gap between interpretable mathematical modelling and adaptive AIbased optimization.

Future work may extend this approach by:

- Incorporating stochastic disturbances more explicitly.
- Modeling multiple interconnected recycling facilities as a network.
- Using real operational data from industrial ewaste plants to calibrate and validate the model.
- Integrating economic and environmental metrics into the optimization objective.

Overall, the proposed framework offers a promising direction for designing intelligent, sustainable, and mathematically grounded ewaste recycling systems.

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