

IMPACT OF CHARGE AND COSMOLOGICAL CONSTANT ON THE GRAVITATIONAL COLLAPSE OF LEMAITRE TOLMAN BONDII SPACETIME IN $f(R,T)$ GRAVITY

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Abstract

The main aim of this paper is to study the Impact of charge on the gravitational collapse of Lemaitre Tolman Bondi (LTB) space time in $f(R, T)$ gravity. We also discuss the role of $f(R, T)$ gravitational collapse as well as electromagnetic field on collapsing bodies. For this purpose, we consider spherically symmetric LTB relativistic body joined with ideal matter distribution along with electromagnetic field in $f(R, T)$ gravity. We explore the exact analytical models of Maxwell $f(R, T)$ field equations with constant Ricci scalar and trace of energy momentum tensor. The dynamics of collapse system is investigated through system energy content, cosmological and black hole horizons in the presence of electromagnetic field. It is concluded that in addition to matter variables and $f(R, T)$ corrections, the charge also affects the time interval between formation of respective horizons and singularities. We also find that $f(R, T)$ corrections as well as electromagnetic field tend to slow down the collapse process.

Key words and phrases. *Electromagnetic field, Relativistic System, $f(R, T)$ modified gravity*

1 Introduction

There are four fundamental forces such as gravitational force, weak nuclear force, electromagnetic force and strong nuclear force among these forces gravitational force is the weakest force. It plays a vital role for studying the large- scale structures due to its long-range nature as compared to other fundamental forces. It helps to study the structure formation of galaxies, black holes and inflation of the universe. General Relativity (GR) is the best theory for the description of gravitational force in terms of geometry of manifold and its matter contents. It solved many hidden mysterious problems since its natal. Yet there are some problematic issues which have to be resolved such as accelerated expansion of universe and cosmological constant problems. It also explains many astrophysical phenomena such as stellar evolution, gravitational collapse and big bang.

Gravitational collapse is the astronomical phenomenon in which a star with mass much larger than the solar mass contracts to a point under the effect of its own gravity. It occurs when internal nuclear fuel of a massive star fails to supply high pressure to balance gravity. According to GR , gravitational collapse of massive objects (having mass = $10^6 M_{\odot} - 10^8 M_{\odot}$, where M is the mass of the Sun) results to the formation of spacetime singularities in our universe. We have

to studied the different facts of gravitational collapse in the form of $f(R, T)$ modified theory of gravity as well as electromagnetic field on collapsing bodies. For this purpose, we consider spherically symmetric LTB relativistic body joined with ideal matter distribution along with electromagnetic field in $f(R, T)$ gravity. We have to explore BHH and CH and study their dynamical properties during collapsing mechanism.

Many papers have been written to investigate the gravitational collapse in extended theories of gravity [1]-[2]. Sharif and Kausar [3] studied the spherically symmetric perfect fluid gravitational collapse in the $f(R)$ theory of gravity. Farasat et al. [4]

investigated collapse with dust in the $f(R)$ theory of gravity. Many theories shows that gravity is a higher dimensional interaction. It would be important to study the gravitational collapse and singularity formation in higher dimensions. Dadhich et al. [5] studied the gravitational collapse in pure Lovelock gravity in higher dimensions. Ghosh and Beesham [6] investigate the higher dimensional inhomogenous dust collapse and cosmic censorship. Patil et al. [7] investigated the naked singularities and structure of geodesic in higher dimensional dust collapse. Sharif and Ahmed [8] studied the higher dimensional perfect fluid collapse with a cosmological constant. Further Samanta and Dhal [9] have constructed higher dimensional cosmological models with a perfect fluid source in $f(R, T)$ gravity. Chaubey and Shukla [10] have found a new class of Bianchi cosmological models in $f(R, T)$ gravity. In consequences of the above studies, section 2 represents spherical LTB solution of field equations after joining inner and outer space time over fixed boundary. We also Analyze how Maxwell- $f(R, T)$ terms affect the collapse rate. In section 3, we evaluate apparent horizons and discuss their dynamical properties. Finally, we summarize the hole discussion in last section.

2 LTB Solutions in $f(R, T)$ Modified Gravity

Let us consider the celestial system under consideration having three-dimensional time like boundary surface, Ω , which separate the line elements into two major interior and exterior parts. The interior portion is represented by a non-static spherically symmetric LTB spacetime which is given as

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{1}$$

where the A and B are functions of t and r . The most general form of interior portion LTB metric given as

$$ds^2 = dt^2 - \frac{B^2}{(h + \epsilon)} dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{2}$$

where ϵ is a constant whose value may be 0 or ± 1 while h is an arbitrary radial function such that $h + \epsilon \geq 0$. This is one of the important model of astrophysics which shows that inhomogeneous spherical non radiating geometry. It has fundamental importance in the study of cosmology, quantum gravity as well as cosmic inhomogeneities. Furthermore, the outside region is described with following line element

$$ds_+^2 = \left(1 - \frac{2M}{\tilde{R}} + \frac{Q^2}{\tilde{R}^2}\right) d\tilde{t}^2 + 2\tau d\tilde{R} - R^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{3}$$

where Q electric charge and M indicates mass of total energy content of the system, while ν represent retarded time. The $f(R, T)$ field equations with Electromagnetism are given as

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) f_R(R, T) = \kappa(T_{\mu\nu} + T_{\mu\nu}^{em}) + f_T(R, T)(T_{\mu\nu} + pg_{\mu\nu}). \quad (4)$$

Where $T_{\mu\nu}$ is the energy momentum tensor for perfect fluid. The energy momentum tensor for perfect fluid is given as

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (5)$$

where p is fluid pressure and ρ is the energy density while $u_\alpha = \delta^0_\alpha$ is the four velocity vector in co-moving coordinates. $T_{\mu\nu}^{em}$ is the energy momentum tensor for electromagnetic field which can be written as

$$T_{\mu\nu}^{em} = \frac{1}{4\pi} (-g^{\delta\omega} F_{\mu\delta} F_{\nu\omega} + g_{\mu\nu} F_{\delta\omega} F^{\delta\omega}), \quad (6)$$

where $F_{\mu\nu} = \frac{d\phi}{dx^\mu} - \frac{d\phi}{dx^\nu}$ is Maxwell field tensor with four potential. Now substituting Eq. (5) and (6) into Eq. (4), the field equations take the form

$$R_{\mu\nu} = 8\pi[(\rho + p)u_\mu u_\nu + \frac{1}{2}(p - \rho)g_{\mu\nu} + T_{\mu\nu}^{em} - \frac{1}{2}T g_{\mu\nu}] + f(T)g_{\mu\nu} - \Lambda g_{\mu\nu} \quad (7)$$

where $f(t) = \lambda(T)$ and Λ is cosmological constant. The Maxwell's field equations in the form of electromagnetic field are given as

$$F^{\alpha\beta} = \gamma_0 J^\alpha, F[\alpha\beta;\gamma] = 0, \quad (8)$$

where $J^\alpha(t, r)$ is Noether current and γ_0 magnetic permeability. The electric charge of the observed system is found to be in rest state due to co-moving coordinates which have zero magnetic field. Thus we have

$$\varphi_\alpha = \varphi(t, r)\delta^0_\alpha, J^\alpha = \mu(t, r)V^\alpha, \quad (9)$$

where μ and φ indicate free charge density and scalar potential respectively. By using the field Eq. (1) and (8) into Eq. (9), we obtain

$$\frac{\partial^2 \varphi}{\partial r^2} - \left(\frac{A}{r} - \frac{2B}{r^2} \right) \frac{\partial \varphi}{\partial r} = 4\pi\mu A, \quad (10)$$

$$\frac{\partial}{\partial r} \left(\frac{\partial \varphi}{\partial t} \right) - \left(\frac{A}{r} - \frac{2B}{r^2} \right) \frac{\partial \varphi}{\partial r} = 0, \quad (11)$$

where dot symbolizes for $\frac{\partial}{\partial t}$ and prime for $\frac{\partial}{\partial r}$. By Integrating Eq. (6), we obtain

$$\varphi^r = \frac{qA}{B^2}, \quad (12)$$

where

$$q(r) = 4\pi \int_0^r \mu A^2 dr, \quad (13)$$

where $q(r)$ is the amount of charge found into the interior portion of LTB spacetime. The uniform charge distribution over unit spherical area is known as the electric field strength which is given as

$$E(t, r) = \frac{q}{4\pi B^2}. \quad (14)$$

For the interior space time Eq.(2) and (9) gives the following non vanishing set of equations

$$R_{00} = -\frac{\ddot{A}}{A^2} - \frac{\ddot{B}}{B} = 4\pi[(\rho + 3p) + 8\pi^2 E^2 + \lambda(\rho - 3p) - \Lambda], \quad (15)$$

$$R_{01} = \frac{2\dot{B}^r}{B} - \frac{2\dot{A}B^r}{AB}, \quad (16)$$

$$R_{11} = \frac{2A\dot{A}\dot{B}}{B} - \frac{2}{B} \left(\frac{A^r B^r}{A} - \frac{2}{A} \right) = (-A) [4\pi(p - \rho) + 160\pi^2 E^2] + \lambda(\rho - 3p) - \Lambda, \quad (17)$$

$$R_{22} = \frac{2}{A} \left(\frac{B\dot{B}}{B} + \frac{2}{A^2} (BB^r + B - \frac{BA^r B^r}{A} - A) \right) = (-B^2) [4\pi(p - \rho) - 8\pi^2 E^2] + \lambda(\rho - 3p) - \Lambda. \quad (18)$$

$$R_{33} = \sin^2 \theta R_{22}. \quad (19)$$

Moreover, in order to find out exact analytical models of spherically symmetric LTB fourth order metric $f(R, T)$ field equations, we confine our analysis for today value of cosmological Ricci scalar. In this context the field equation (15)-(21) reduce to

$$R_{00} = -\frac{\ddot{A}}{A^2} - \frac{\ddot{B}}{B} = 4\pi[(\rho + 3p) + 8\pi^2 E^2 + \lambda(\rho - 3p) - \Lambda], \quad (22)$$

$$R_{01} = \frac{2\dot{B}^r}{B} - \frac{2\dot{A}B^r}{AB} = 0, \quad (23)$$

$$R_{11} = \frac{2}{A^2} \left(\frac{B^r}{B} - \frac{A^r B^r}{AB} \right) - \frac{\ddot{A}}{A^2} - \frac{\dot{A}\dot{B}}{AB} = [4\pi(p - \rho) + 160\pi^2 E^2] + \lambda(\rho - 3p) - \Lambda, \quad (24)$$

$$\frac{1}{A^2} \left(\frac{B^r}{B} - \frac{A^r B^r}{AB} \right) - \frac{\ddot{A}}{A^2} - \frac{\dot{A}\dot{B}}{AB} = 0, \quad (25)$$

$$\begin{aligned}
 \ddot{B} \quad \dot{B} \quad \dot{B} \quad \dot{A} \\
 R_{22} = \frac{1}{A^2} + \left\{ \frac{B}{B} - \frac{A}{A} - \frac{B^2}{B^2} \right\} - \frac{B}{B} \left(\frac{B}{B} + \frac{A}{A} \right) \\
 \left(\frac{B}{B} \right)
 \end{aligned} \tag{26}$$

$$= [4\pi(p - \rho) - 8\pi^2 E^2] + \lambda(\rho - 3p) - \Lambda.$$

$$R_{33} = \sin^2 \theta R_{22}. \tag{27}$$

Integration of Eq.(23)
 gives

$$A = \frac{B^r}{\eta}, \quad (28)$$

where $\eta = \eta(r)$ is an integration constant. By substituting the value of A in Eqs.(22)-(27) and after doing some simple steps, we obtain the following equation

$$\frac{B \dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} - (\eta^2 - 1) \frac{1}{B^2} = 4\pi (2p - 21E^2) + \lambda(\rho - 3p) + \Lambda. \quad (29)$$

Moreover, in order to find the solution of last equation, we assume the pressure and density as constant quantities i.e., $p = p_c$ and $\rho = \rho_c$. The integration of Eq.(29) yields

$$B \dot{B}^2 = (\eta^2 - 1) + \frac{-21E^2}{c} + \frac{2}{c} \frac{2m}{B} \{4\pi^2(2p_c - \lambda(\rho_c - 3p_c) + \Lambda)\} B + B, \quad (30)$$

where $m = m(r)$ has a correspondence with the mass function of the collapsing celestial *LTB* stellar body which is assumed to be positive. Now putting $\eta = 1$ in Eq.(28), we obtain

$$B \dot{B}^2 = \frac{-21E^2}{c} + \frac{2}{c} \frac{2m}{B} \{4\pi^2(2p_c - \lambda(\rho_c - 3p_c) + \Lambda)\} B + B. \quad (31)$$

By using (23), (28) and (30), it follows that

$$m^r = (\rho_c + 48\pi^2 E^2 - \frac{1}{3} p_c) B^2 B^r. \quad (32)$$

By integrating (32) with respect to r yields

$$m(r) = (\rho_c + 48\pi^2 E^2 - \frac{1}{3} p_c) \int_0^r B^2 B^r dr + \zeta(t). \quad (33)$$

Where $\zeta(t)$ is an integration function. For matching inner and outer geometries over fixed boundary surface Ω in $f(R, T)$ theory, we use matching criteria which is given as

$$M_{\Omega} = \frac{Q^2}{2} (1 + \frac{Q^2}{B^2} - \frac{1}{A^2}), \quad (34)$$

Now by Using Eq.(28) and (30) into (34), we get

$$M_{\Omega} = \frac{Q^2}{2} \left[\frac{1}{B^2} + m(r) + \frac{2}{2} (2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda \right] B^3. \quad (35)$$

The Misner-Sharp mass function shows energy content of spherically symmetric metric [11], which is given as

$$m(t, r) = \frac{R}{2} (1 - g^{\mu\nu} R_{,\mu} R_{,\nu}), \quad (36)$$

This is the mass function for *LTB* geometry in the form of electromagnetic field

$$m(t, r) = \frac{B}{2} \left\{ 1 + \frac{B^r}{A} \right\} + \frac{q^2}{2B}. \quad (37)$$

In view of Eq.(28) and (30), Eq.(37) becomes

$$M = \frac{q^2}{2B} + m_0(r) + \frac{(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda}{\{4\pi\}} + \frac{B^3}{2}. \quad (38)$$

By using Eq.(28) and (30) the exact analytical solution of the field equations can be written as

$$m^r A = - \frac{1}{3} \frac{84(\pi^2 (2p_c - 21E^2) - \lambda(\rho_c - 3p_c) + \Lambda)}{2} \frac{2m}{2} \frac{1}{3} \quad (39)$$

$$\left[\frac{1}{3} \sinh^3 \delta(t, r) + 3 \sinh \delta(t, r) \cosh \delta(t, r) \{63(\pi E^2) - \lambda(\rho_c - 3p_c) + \Lambda\} \right], m$$

and

$$B = \left(\frac{2m}{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda} \right)^{\frac{1}{3}} \times \sinh^3 \delta(t, r), \quad (40)$$

where

$$\delta = \frac{1}{2} \sqrt{\frac{1}{2} [4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda] \times [t_0(r) - t]},$$

$t_0(r)$ is singularity formation of any specific shell with radial coordinate r . We have assumed positivity of the term $\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}$ with $\eta = 1$ to evaluate above solution.

3 Apparent Horizons

In this section, we analyze surface of apparent horizon in LTB spacetime. Two trapped spheres whose outward normals are null is used to find apparent horizons. Also, we discuss the physical significance of apparent horizons and singularity. The general expression to calculate such a fixed boundary can be given as

$$g^{\mu\nu} B_{,\mu} B_{,\nu} = B \frac{B^{r2}}{A^2} = 0. \quad (41)$$

By using Eq.(28) and (30) into (41), we obtain

$$\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\} B^3 - 3B + 6m = 0. \quad (42)$$

Which is cubic equation in the form of B , that gives the values of apparent horizons. Putting assumption $\Lambda = -[4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda]$ in Eq.(42), yields $B = 2m$ which is known as Schwarzschild horizon. Now, we discuss the following cases by considering the positive roots of Eq.(42)

3.1 Case-I

when

$$3m < \frac{s}{\frac{1}{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}},$$

we obtain two horizons B_c and B_{bh} , which indicates cosmological horizon and black hole horizon, given as

$$B_c = \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{3} \cos \frac{\varphi}{3}, \quad (43)$$

$$B_{bh} = \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{3} (\cos \frac{\varphi}{3} - \sqrt{3} \sin \frac{\varphi}{3}), \quad (44)$$

where

$$\cos \varphi = \frac{3m}{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}. \quad (45)$$

For particular case if $m = 0$ in Eq.(43), then Eqs.(43) and (44) take the form

$$B_c = \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{3}, \quad (46)$$

and

$$B_{bh} = 0 \quad (47)$$

3.2 Case-II

When

$$3m = \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{2},$$

we obtain a unique solution

$$B_c = B_{bh} = \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{2}. \quad (48)$$

This indicates both horizons coincide at a point. The range of CH and BHH can be written as

$$0 \leq B_{bh} \leq \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{2} \leq B_c \leq \frac{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}{2}. \quad (49)$$

The maximum proper area of BHH is given as

$$4\pi B^2 = \frac{8\pi}{\sqrt{\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}}$$

The minimum area of cosmological horizon CH lies between

$$\dots \quad (50)$$

and

$$\frac{\{4\pi^2(2p_c - 21E^2) + \dots\}}{\lambda(\rho_c) \dots - 3p_c + \Lambda} \quad (51)$$

$$\frac{24\pi}{\{4\pi^2(2p_c - 21E^2) + \dots\}} \quad (52)$$

3.3 Case-III

when

$$s = \frac{1}{3m} >$$

$$\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}$$

which leads to zero number of positive roots which suggest null apparent horizon. To evaluate time require by collapsing system for the apparent horizon formation we have to used Eq.(40) and (42)

$$t_n = -\sqrt{\frac{\frac{\sqrt{2}}{2}}{3\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}}} \sinh^{-1} \sqrt{\frac{B_n}{2m} - 1}, \quad n = (1, 2). \quad (53)$$

In the limiting case when

$$\Lambda \rightarrow \{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\},$$

then Eq.(53) leads to

$$\frac{B_n}{2m} = \cosh^2 \delta, \quad (54)$$

where

$$\delta = \frac{1}{2} \sqrt{3\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\} [t_0(r) - t_n]}.$$

It is clear from Eq.(49) and (53) that $B_c \geq B_{bh}$ and $t_{bh} \geq t_c$, respectively. Here t_{bh} and t_c represent the time formation cosmological and black hole horizons respectively. The condition $t_{bh} \geq t_c$ shows that CH appears earlier to BHH. The time difference between the formation of cosmological horizon and singularity formation also between black hole horizon BHH and singularity can be found from Eq.(43)-(45).

$$\left(\frac{B_c}{2m}\right) \frac{1}{3} \left(\frac{-\sin \varphi}{\sin \varphi} + \frac{3 \cos^2 \varphi}{\cos \varphi}\right) < 0, \quad (55)$$

$$\left(\frac{B_{bh}}{2m}\right) \frac{1}{3} \left(\frac{-\sin \varphi}{\sin \varphi} + \frac{3 \cos^2 \varphi + 4\pi}{\cos \varphi}\right) > 0. \quad (56)$$

The time difference between the singularity and apparent horizon is

$$T_n = t_0 - t_n. \quad (57)$$

It follows from Eq.(54) and (57) that

$$\frac{dT_n}{d\left(\frac{B_n}{2m}\right)} = \frac{1}{(\cosh \delta \sinh \delta) \left(\frac{1}{3\{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda\}} \right)}. \quad (58)$$

Applying chain rule we obtain the time interval variation with respect to time, after

using Eq.(55) and (58) we get

$$\frac{dT_c}{dm} = \frac{1 - 3p + \Lambda \cosh \delta}{\lambda(\rho) \sqrt{4\pi^2(2p - 3)}} \left(\frac{-\sin \frac{\varphi}{3}}{\sin \varphi} + \frac{3 \cos \frac{\varphi}{3}}{\cos \varphi} \right) < 0, \quad (59)$$

where T_c is a decreasing function of m which shows that the value of system mass increases. Time interval between singularity and CH decreases. Thus the time interval between these two events is inversely proportional to the energy content of the celestial collapsing object. By making the use of Eq.(56) and (58), we obtain

$$\frac{dT_{bh}}{dm} = \frac{1}{\sqrt{\frac{21E^2}{c^2} + \frac{3p}{c} + \Lambda} \cosh\delta \sinh\delta} \times \left(\frac{-\sin\frac{\varphi+4\pi}{3}}{3} + \frac{3\cos\varphi}{3} \right) > 0. \quad (60)$$

This indicate that T_{bh} is an increasing function of m which implies with increase of time. This also shows that the time difference between the formation of black hole horizon and singularity increases with increase in time.

4 Summary and Discussion

We studied the charged Adiabatic LTB gravitational collapse by taking non-static spherically symmetric LTB spacetime as interior portion and schwarzschild as exterior portion. We obtained two physical horizon namely cosmological horizon and black hole horizon. It is shown that time formation of black hole horizon is greater than of cosmological horizon. Also both horizons are formed earlier than the singularity. This shows that singularity is covered and $f(R, T)$ theory of gravity supports CCC . Now we discuss the effect of electromagnetic field on the collapse rate. As it is known that the Newtonian potential for the geometry exterior at Ω is given as

$$\chi = (1 - g_{00})^{\frac{1}{2}}. \quad (61)$$

For the exterior metric Eq.(61) can be evaluated as

$$\chi = \frac{2m}{R} + \frac{B^2}{6} \sqrt{\frac{21E^2}{c^2} + \frac{3p}{c} + \Lambda}. \quad (62)$$

We can calculate the value of Newtonian force, F , by taking derivative of the above equation as

$$F = -\frac{2m}{R^2} + \frac{1}{3} \frac{d}{dR} \left(\frac{21E^2}{c^2} + \frac{3p}{c} + \Lambda \right). \quad (63)$$

Where the quantities m and R over the hypersurface have significant importance in the collapsing rate of the observed system. The effects of Newtonian force can be analyze by taking different values of the parameters which is following as.

For $m = \frac{\sqrt{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda}}{3}$ and $R = \frac{\sqrt{4\pi^2(2p_c - 21E^2) + \lambda(\rho_c - 3p_c) + \Lambda}}{3}$, we obtain

$F = 0$. This shows that Newtonian force has null effect on the collapse of adiabatic charged

spherical star, if m and R both are able to attain such values during evaluation process.

- The Newtonian force will impart repulsive effects on the collapsing ideal fluid only if $F > 0$. This could be possible if $m > \frac{1}{6 \sqrt{\{4\pi^2(2\rho_c - 21E^2) + \lambda(\rho_c - 3\rho_c) + \Lambda\}}}$ and $R > \frac{1}{\sqrt{\{4\pi^2(2\rho_c - 21E^2) + \lambda(\rho_c - 3\rho_c) + \Lambda\}}}$.

m and R will obey these inequalities if $1 < 4\pi\{(\rho + p) + 23\pi E^2(t, r)\}$. This shows significance of Ricci scalar corrections and electromagnetic field to determining the rate of collapse.

Above mentioned constraints will support the quick collapse rate. However, this is not possible in our study, since we require $\{4\pi(2p_c - 21E) + \lambda(\rho_c - 3p_c) + \Lambda\} > 0$ as mentioned earlier.

Finally we can summarize the outcome of thesis as follows: Eq.(63) shows that $f(R, T)$ dark source correction of collapsing mechanism thereby improving stability of the system in cosmos which is consistent with (26)-(61). The equation (49) and (53) shows that time require to appear BHH is greater than that of CH . It is noticed that electromagnetic field tends to decrease time formation of BHH and CH . We can see from equations (59) and (60) that horizons emerge prior to singularity even in the presence of electromagnetic field which shows the start of covered singularity. This confirms the validity of cosmic censorship conjecture in Maxwell $f(R, T)$ theory of gravity.

References

- [1] Maeda, H. *Phys. Rev. D* **2006**, 73, 104004(1)-104004(17)
- [2] Ghosh, S.G.; Mahraj, S.D. *Phys. Rev. D* **2012**, 85, 124064(1)-124064(9).
- [3] Sharif, M.; Kausar, H.R. *Astrophys. Space Sci.* **2011**, 331, 281-288
- [4] Farasat Shamir, M.; Ahmad, Z.; Raza, Z. *Int. J. Theor. Phys.* **2015**, 54, 1450-1460.
- [5] Dadhich, N.; Ghosh, S.G.; Jhing, S. *Phys. Rev. D* **2013**, 88, 084024(1)-084024(8).
- [6] Ghosh, S.G.; Beesham, A. *Phys. Rev. D* **2001**, 64, 124005(1)-124005(5).
- [7] Patil, K.D.; Ghate, S.H.; Saraykar, R.V. *Indian J. Pure Appl. Math.* **2002**, 33, 379-389.
- [8] Sharif, M.; Ahmad, Z. *Acta Phys. Polon. B* **2008**, 39, 1337-1347.
- [9] G.C. Samanta, S.N. Dhal **2013** *Int. J. Theor. Phys.* 52 1334
- [10] R. Chaubey, A.K. Shukla **2013** *Astrophys. Space Sci.* 343 415.
- [11] Misner, C.W., Sharp, D.: *Rev.* **136**, B571 (1964)