ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
CJSSR
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
Vol.03 No.04 (2025)

SCHEDULING A GLOBAL FOOTBALL LEAGUE: A MATHEMATICAL FRAMEWORK FOR COMPETITIVE BALANCE AND LOGISTICAL EFFICIENCY

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Abstract

This article provides a mathematical model of the optimization of schedules, competitive balance and organization of an international football league, hereafter referred to as the Global Sports League (GSL) In the initial phase, we conducted research to identify the critical factors that influence scheduling, such as team rankings, geographical locations, and revenue considerations. We then formalized these factors into a set of constraints and objectives, ensuring a mathematically robust approach. Instead of manually weighting factors, we utilized a hierarchical decision-making process to systematically determine their relative importance.

For model validation, we tested our approach on past seasons, analyzing the fairness and efficiency of our generated schedules compared to historical ones. This step allowed us to refine weight coefficients and further improve our methodology.

To generalize our model for different sports within the GSL, we adapted our approach to accommodate variations in game frequency, venue availability, and international participation. We introduced sport-specific constraints and adjusted our weighting matrices accordingly to enhance adaptability across various league formats.

Next, we constructed compatibility matrices using statistical distributions to evaluate match-up fairness and competitive balance. By analyzing past performance trends and team dynamics, we ensured that the scheduling algorithm fosters engaging and unpredictable league outcomes.

For visualization, we implemented radar charts to compare different scheduling structures, with vertices representing critical scheduling factors. By analyzing the overlap between different schedule structures, we quantitatively assessed their effectiveness in achieving fairness and minimizing logistical burdens. Finally, we projected future scheduling adaptations by evaluating potential changes in league composition, team performances, and travel constraints. By applying trend analysis over different time frames, we estimated long-term scheduling impacts and optimized the model for evolving league dynamics. Our model offers a structured and flexible approach to scheduling optimization, ensuring competitive fairness and logistical feasibility for the GSL. With further refinement and real-world data integration, it holds the potential to revolutionize league scheduling processes.

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Introduction

The scheming of a world football league is an intricate optimization issue taking into account rivalry, logistical, and commercial limitations. A global event, as opposed to a local one, has to take into consideration wide geographic, team, travel, and broadcasting differences. The interaction between these factors renders them to be inadequately scheduled manually or through heuristics which requires a defined mathematical framework.

Through a comprehensive analysis of various league formats, competition structures, and team dynamics, our team has developed a model that ensures fairness, competitiveness, and financial sustainability. Our approach considers factors such as scheduling efficiency, team rankings, and revenue distribution to create an optimal league framework that benefits all stakeholders. The key issue of global scheduling is to achieve a balance between competitive equality and operational restrictions. The teams should be matched with the opponent of equal strength, be given rest after playing a match and should not have to travel too much as it can undermine their performance. At the same time, the league should be able to stay commercially viable by maximizing the match schedule on a cross-time zone basis and large-demand matches. The current models of sports scheduling, such as round-robin basis, minimizing travel, or Elo-like competitive rating, offer some of the solutions but cannot fulfill the whole range of global logistical and competitive issues at the same time.

Our solution is capable of simulating different league structures and providing insights into the best possible configurations. However, we acknowledge that real-world implementation requires expert validation and refinement. We encourage IMMC-A professionals to collaborate with us in refining the model, incorporating real-time data, and fine-tuning parameters for enhanced accuracy. Additionally, our model is adaptable, allowing adjustments based on unforeseen circumstances such as team withdrawals or scheduling conflicts. This flexibility ensures that the league remains resilient while preserving its core principles of fairness and competitiveness. This paper establishes a combination optimization model to build a schedule of the sports league on the Global Sports League (GSL). The model includes a few of its main elements: (i) a team distribution mechanism (OPTA Power Rankings and randomised pairing to balance the strength of teams); (ii) a match-scheduling system that limits the number of rest breaks, home/away allocation, travel minimisation and time-zone alignment; and (iii) a dynamic rating system (inspired by the Elo model) to track team performance and seeding at the knockout stage. The framework also explains conflicts in international competition, regional differences, and considerations of viewers.

We each make four contributions. Firstly we model the scheduling problem as a multi-objective optimization problem that is based on hierarchical decision-making in order to prevent arbitrary weighting of the constraints. Second, we combine competitive balance, logistical efficiency, and commercial into a single mathematical framework. Third, we test the flexibility of the model with a league-expansion scenario, which raises the number of teams in the league by 24, to 20, which shows that the model is stable with altered densities of fixtures. Lastly, we use the paradigm of the methodology to generalize the system to other team sports by modulating the frequency of matches, setting venue limits, and the dynamic of fatigue.

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This framework is the method that offers a rigorous and extensible method of making the league in the whole world, and a tool that helps the decision makers to make an informed but feasible scheduling system that is also fair.

Methodology

Identifying the Key Factors

Sport Selection and Team Composition

The Global Sports League (GSL) has chosen football (soccer) as the primary sport for its league due to several compelling reasons. Football is the most widely played and followed sport worldwide, making it the ideal candidate for a truly global league. The sport's universal appeal, structured domestic and international competitions, and deeply embedded cultural significance ensure a strong foundation for sustained success. Unlike many other sports, football offers a well-established club system across continents, facilitating a seamless integration of teams from different regions. Moreover, football's relatively low barrier to entry enables widespread participation, fostering inclusivity and diversity within the GSL. By leveraging the sport's extensive reach, the GSL can attract audiences from all over the world, ensuring high levels of engagement, commercial viability, and competitive integrity.

Justification for Selecting Clubs Over National Teams

There are multiple strategic reasons why the GSL has opted to feature club teams instead of national teams, ensuring a more competitive and sustainable league structure:

- **Player Commitment and Interest:** Footballers exhibit greater commitment and consistency when representing their club teams. Clubs provide a stable environment for players to train and develop tactical chemistry, whereas national teams are assembled only periodically. The limited preparation time for national teams often results in less cohesive gameplay, reducing the overall quality of competition.
- **Diversity in Representation:** Selecting club teams ensures participation from a wider range of cities and regions, transcending national boundaries. This approach allows strong football markets to be well-represented while also providing opportunities for emerging footballing regions to compete at a high level. Unlike national teams, which are restricted by nationality, clubs can recruit top players from all over the world, enhancing the competitive nature of the league.
- **Stability and Tactical Cohesion:** Club teams operate throughout the year with consistent squad development and tactical strategies, ensuring higher levels of cohesion and long-term planning. National teams, on the other hand, often experience fluctuations in squad selection, coaching changes, and inconsistent player availability, leading to unpredictable performances. A club-based league structure enables the GSL to maintain a high standard of play with well-defined playing styles and rivalries.
- **Scheduling Flexibility:** Club football operates within a structured seasonal format, allowing the league to optimize match scheduling efficiently. This enables better coordination of fixtures, travel logistics, and player workload management. National team tournaments, in contrast, are restricted to specific international breaks, limiting the flexibility to organize a long-term league competition.

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- **Established Fan Bases and Marketability:** Clubs have deeply rooted fan cultures and strong global followings, making them commercially attractive. These teams have existing brand identities, dedicated supporters, and well-established revenue streams, ensuring financial stability for the league. National teams generate patriotic enthusiasm but may lack year-round engagement, making them less suitable for a sustainable league model.

Team Composition and Selection Criteria

The Global Sports League (GSL) has twenty clubs that are spread in different continents to make it fair in representation as well as competition. The league format is designed to unite existing footballing giants on the one hand, with the new teams on the other, encouraging active international interaction:

- **Europe:** 7 teams
- **South America:** 3 teams
- **North America:** 3 teams
- **Asia:** 3 teams
- **Africa:** 2 teams
- **Oceania:** 2 teams

These clubs have been chosen based on a rigorous appraisal system that is based on the following criteria:

- **Performance and Competitiveness:** Clubs are selected based on their European leagues (e.g., English Premier League, La Liga, Bundesliga) and international competitions of clubs (e.g., the UEFA Champions League, the Copa Libertadores). This criterion will ensure only the most competitive teams participate, hence maintaining a high level of play.
- **Fan Base and Global Appeal:** Clubs that have a large international fan following are preferred because they help the league to succeed commercially and increase audience. The availability of internationally established clubs increases the viewership and sponsorship opportunities.
- **Historical and Cultural Significance:** Teams with rich footballing culture and historical success bring prestige to the league. These clubs create a system of rivalry, mythical reputations and an entire footballing personality that adds to the overall story of the tournament.
- **Financial Stability and Infrastructure:** To be financially viable is very important to the sustainability of the league. The teams that are being selected must possess good financial support, an up-to-date training centre and top-quality stadia to match operations standards in the league.
- **Geographic and Continental Balance:** The league aims at bringing out equal representation in all continents, hence facilitating the growth of football worldwide. This equilibrium counteracts regional superiority and makes fans of the leagues all over the world have a vested interest.
- **Logistical Feasibility:** The factor of travel and infrastructure is an important criterion in team selection. The clubs should be located in areas where there is a well-built transport system that would ensure that a person can travel across the match venues easily.
- **Youth Development and Sustainability:** Preference is given to the clubs which invest in youth academies and developing players on a long-term basis. An investment in developing the new talent is a promise of sustainability and competitiveness of the league.

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Key Aspects in League Scheduling

The league schedule of the GSL should pay keen attention to various critical factors to maximise fair play, competition, and economics:

- **Team Rest and Recovery:** It is necessary to have sufficient rest periods between matches in order to avoid fatigue and injuries among players. Having a well organized schedule would provide players with optimal performance throughout the season.
- **Home/Away Game Distribution:** The teams should be given the same number of home and away matches to maintain fairness. This prevents teams from gaining an undue advantage through an imbalanced schedule
- **Travel Time and Costs:** It is important to reduce unnecessary travelling that would put the players in jeopardy and make the whole process uneconomical. The schedule has also taken into account travel-efficient fixtures, which have decreased long-haul flights and fatigue.
- **Competition and Rivalries:** Traditional rivalries and marquee matchups must be strategically placed within the schedule to maximize fan excitement and maintain engagement throughout the season.
- **Weather and Seasonal Conditions:** The league takes into consideration the seasonal weather conditions, so that the matches are held in the most favourable conditions. The occurrence of extreme weather can necessitate adjustments to match venues or timing.
- **Time Zone Differences:** International scheduling should be able to accommodate the time zones to ensure they get the maximum audience. The games will be played at the most favourable times to ensure the maximum international audience is reached.
- **Competitive Balance:** The fixture list is structured to provide a fair distribution of difficult and easier matches among all the teams will guarantee a perfect balance of competition.
- **International Competition Conflicts:** Teams involved in continental or intercontinental tournaments must have their schedule realigned in order to avoid fixture congestion and player fatigue.
- **Fan Engagement and Regional Preferences:** Match times and places are optimised in such a manner to create maximum attendance at the stadium and TV viewing which coincides with the regional fans.
- **Broadcast and Media Rights:** Schedule should be consistent with broadcasting agreements in order to get maximum exposure and generate revenue.

This structured approach to scheduling ensures that the GSL remains a fair, competitive, and commercially successful global football league.

Division of Teams into the 2 tables

To ensure competitive balance in the Global Sports League (GSL), the 20 teams are divided into two groups of 10 using the OPTA Power Rankings. Teams are ranked based on performance metrics, then paired consecutively (e.g., 1st with 2nd, 3rd with 4th). Each pair is then randomly assigned to Group A or Group B, ensuring an even distribution of team strength. This method prevents one-sided group stages and ensures fair competition. Additional factors like geographical distribution and rivalries may refine allocations for travel efficiency and commercial appeal. By combining data-driven ranking with randomized assignment, the GSL guarantees a balanced and engaging competition.

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GSL Rating Point System

The Global Sports League (GSL) employs a dynamic rating system inspired by the Elo rating model, ensuring a fair and adaptive ranking of teams throughout the competition. Unlike a fixed-point system, this model adjusts points based on the relative strengths of competing teams. Each team starts with 100 rating points, and their ratings fluctuate after every match depending on the result and the opponent's strength. A team defeating a higher-rated opponent gains more points, while a loss to a weaker team results in a greater rating drop. This ensures that team ratings accurately reflect performance rather than just win-loss records.

The system balances competitiveness and ranking stability by adjusting point exchanges based on rating differences. If two equally rated teams play, a fixed number of points is exchanged. However, as the rating gap widens, the point exchange scales—underdogs gain more for victories, while favorites lose more in upsets. This prevents ranking stagnation and ensures that consistent performance dictates standings. By the end of the league phase, top-rated teams from each group advance to the knockout stage, making every match impactful while maintaining fairness and competition across the tournament.

Mathematical Model for the GSL Schedule

Group Division Based on OPTA Power Rankings

To ensure competitive balance within the Global Sports League (GSL), the 20 participating teams are divided into two groups of 10 teams each. This division is guided by the following methodology:

Utilizing OPTA Power Rankings

The GSL leverages the OPTA Power Rankings—a comprehensive global team ranking system that assigns an ability score to over 13,000 men's domestic football teams on a scale from zero to 100—to assess team strengths. These rankings are updated daily and provide a standardized metric for comparing teams across different leagues and continents.

Ranking and Pairing Process

The 20 teams are ranked according to their OPTA Power Rankings, from highest to lowest. Subsequently, teams are paired based on their consecutive rankings to form 10 pairs:

- Pair 1: Teams ranked 1 and 2
- Pair 2: Teams ranked 3 and 4
- Pair 3: Teams ranked 5 and 6
- Pair 4: Teams ranked 7 and 8
- Pair 5: Teams ranked 9 and 10
- Pair 6: Teams ranked 11 and 12
- Pair 7: Teams ranked 13 and 14
- Pair 8: Teams ranked 15 and 16
- Pair 9: Teams ranked 17 and 18
- Pair 10: Teams ranked 19 and 20

Random Assignment to Groups

To distribute the paired teams equitably between the two groups, each pair undergoes a random assignment process:

- One team from each pair is randomly assigned to Group A.
- The other team from the same pair is assigned to Group B.

This randomization ensures that each group receives a mix of teams with varying strengths, promoting competitive fairness.

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CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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Mathematical Representation of Group Assignment

Let S_n represent the team ranked n in the OPTA Power Rankings, where n ranges from 1 to 20. Define G_m as the m-th pair of teams:

$$G_m = \{S_{2m-1}, S_{2m}\}, \quad m = 1,2,\dots,10$$

For each pair G_m, a random variable Y_m is introduced, where:

- $Y_m = 0$ assigns S_{2m-1} to Group X and S_{2m} to Group Y. - $Y_m = 1$ assigns S_{2m-1} to Group Y and S_{2m} to Group X.

The final group assignments are given by:

$$\text{Group X} = \{S_{2m-1} \mid Y_m = 0\} \cup \{S_{2m} \mid Y_m = 1\}$$
$$\text{Group Y} = \{S_{2m-1} \mid Y_m = 1\} \cup \{S_{2m} \mid Y_m = 0\}$$

This ensures an even distribution of team strength across both groups while maintaining fairness and competitiveness.

Consideration of Additional Factors

While OPTA Power Rankings provide a robust framework for assessing team strength, the GSL also considers other factors to refine group assignments:

- **Geographical Distribution:** Ensuring a diverse representation of teams from different continents within each group to enhance global appeal and logistical feasibility.
- **Historical Rivalries:** Acknowledging traditional rivalries and aiming to distribute them across groups to maintain spectator interest throughout the league stages.
- **Marketability:** Balancing teams with substantial fan bases and commercial value between groups to optimize viewership and sponsorship opportunities.

By integrating these considerations with the OPTA-based pairing and randomization process, the GSL strives to create a fair, exciting, and commercially viable competition.

Fundamentals of League Scheduling

To construct a fair and practical schedule for the Global Sports League (GSL), we must ensure:

1. Fair distribution of games across teams
2. Equitable travel time and distance
3. Balanced competitiveness of matchups
4. Team rest and recovery
5. Consideration of international competitions
6. Fan engagement and media rights

Our mathematical model incorporates these elements while ensuring a sustainable and competitive league.

Game Distribution and Match Scheduling

The 20 teams are divided into two groups of 10, where each team plays every other team in their group twice (home and away), resulting in 18 total matches per team. Define $X_{ijt} \in \{0,1\}$ as a binary variable indicating whether team i plays against team j at time t at home. The constraints ensuring fair scheduling are:

- Each team plays 18 matches:

$$\sum_{t=1}^{22} \sum_{j=1, j\neq i}^{10} X_{ijt} = 18, \quad \forall i.$$

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- A balanced home-away schedule:

$$\sum_{t=1}^{22}\sum_{j=1,j\neq i}^{10} X_{ijt} = 9, \quad \sum_{t=1}^{22}\sum_{j=1,j\neq i}^{10} X_{jit} = 9, \quad \forall i.$$

- No team plays more than one match per week:

$$\sum_{j=1,j\neq i}^{10} X_{ijt} + X_{jit} = 1, \quad \forall i, t.$$

Additionally, a rolling fixture algorithm is introduced to ensure that teams do not have consecutive home or away games for extended periods, ensuring fairness.

Rest, Recovery, and Travel Efficiency

Player fatigue impacts performance, necessitating adequate rest between matches. Define R_i as the rest factor for team i:

$$R_i = \sum_{t=1}^{22}\left(\sum_{j=1,j\neq i}^{10} X_{ijt} + \sum_{j=1,j\neq i}^{10} X_{jit}\right) W_t,$$

where W_t is a binary indicator of whether a break week exists. Optimizing:

$$\max\min_i R_i.$$

Break weeks are placed after every four matchweeks:

$$B_t = 1, \quad \forall t \in \{5,10,15,20\}.$$

This ensures player recovery and prevents overloading squads competing in multiple tournaments.

Travel Optimization

Travel efficiency is crucial for minimizing fatigue. Define A_{ij} as the distance between teams i and j, and D_i as the travel burden:

$$D_i = \sum_{t=1}^{22}\sum_{j=1,j\neq i}^{10} A_{ij} X_{ijt}.$$

The objective is to minimize total travel:

$$\min \sum_{i=1}^{10} D_i.$$

International travel is restricted to twice every five weeks:

$$\sum_{t=5k+1}^{5k+5}\sum_{j=1,j\neq i}^{10} A_{ij} X_{ijt} \leq 2D_{\max}, \quad \forall k, \forall i.$$

A geographically optimized clustering algorithm groups teams based on distance constraints, ensuring the fewest long-haul trips while maintaining fixture fairness.

Competitive Balance and Fair Matchups

Define C_i as the cumulative ranking strength of opponents faced by team i:

$$C_i = \sum_{t=1}^{22}\sum_{j=1,j\neq i}^{10} R_j X_{ijt},$$

where R_j is the dynamic rating of team j. To balance competitiveness, we impose:

$$\max\left(\max_i C_i - \min_i C_i\right) \leq \epsilon.$$

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Additionally, historic match data is incorporated to ensure that no team has an excessive run of difficult games in a short span.

Time Zone Adjustments and Fan Engagement

To maintain global viewership, match scheduling accounts for time zones. Define Z_t as the number of teams playing at an inconvenient local time:

$$Z_t = \sum_{i=1}^{10} \mathbb{I}\,(T_i \notin [T_{opt} - \Delta, T_{opt} + \Delta]),$$

where T_i is the local match time, T_{opt} is the preferred broadcasting time, and Δ is the acceptable range. Optimization aims to:

$$\min \sum_{t=1}^{22} Z_t.$$

An adaptive scheduling model dynamically adjusts kickoff times based on regional viewership peaks.

Impact of International Competitions and Media Rights

Teams competing in multiple tournaments need flexibility. Define I_i as an indicator for whether team i is playing in another tournament that week:

$$I_i = \begin{cases} 1, & \text{if team } i \text{ has an international fixture} \\ 0, & \text{otherwise.} \end{cases}$$

To prevent congestion:

$$\sum_{t=1}^{22} I_i\, X_{ijt} \le \tau, \quad \forall i,$$

where τ is the maximum allowable overlap. This ensures minimal fixture congestion while maximizing club participation.

GSL Point System and Rating Model

Mathematical Framework for Rating Adjustments

Each team starts with an initial rating of $R_0 = 100$. After every match, the ratings are updated based on the expected and actual results.

Expected Win Probability

For two teams A and B with ratings R_A and R_B, the expected probability of A winning against B is given by:

$$P_A = \frac{1}{1 + 10^{\frac{R_B - R_A}{400}}}, \quad P_B = \frac{1}{1 + 10^{\frac{R_A - R_B}{400}}}$$

Rating Update Function

After the match, the new ratings are updated as:

$$R'_A = R_A + K(S_A - P_A), \quad R'_B = R_B + K(S_B - P_B)$$

where:

- S is the actual match result, with:

$$S = \begin{cases} 1, & \text{win} \\ 0.5, & \text{draw} \\ 0, & \text{loss} \end{cases}$$

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- P_A and P_B are the expected probabilities.
- K is the adjustment factor, varying with rating differences.

Adjustment of the K-Factor

To ensure fair point distribution, K is dynamically adjusted based on the rating gap:

$$K = K_0\left(1 + \frac{|R_A - R_B|}{800}\right), \quad K_0 = 10$$

where:

- A larger rating gap results in a higher K, increasing rating volatility.
- A smaller gap keeps K stable, ensuring gradual rating shifts.

Group Phase Application

Teams are divided into two groups of 10. Ratings are updated after every match within the group. At the end of the group stage:

- The top four teams from each group qualify for the knockout stage.
- Knockout stage matchups are seeded based on the final ratings.

Mathematical Properties of the Model

The rating system satisfies the following constraints:

- **Total rating conservation:** Over many matches, rating gains and losses tend to balance out.
- **Reward for upsets:** If $R_A < R_B$ and A wins, then $|R'_A - R_A| > |R'_B - R_B|$.
- **Long-term stability:** As teams play more matches, the probability distributions stabilize around performance-based values.

Knockout Stage and Rating Influence

In the knockout stage:

- Higher-rated teams face lower-rated qualifiers.
- The rating difference at the end of the season predicts the probability of winning the final.

The probability P_{win} of a team winning the final match, given ratings R_1 and R_2, is:

$$P_{win} = \frac{1}{1 + 10^{\frac{R_2 - R_1}{400}}}$$

This function ensures that the final match remains competitive while favoring the stronger team.

Final Remarks

The GSL rating system balances fairness, reward for strong performance, and unpredictability by adjusting ratings dynamically based on match outcomes and relative strengths.

Knockout Round System

After the league phase, the top four teams from each group advance to the knockout stage. The elimination round follows a single-elimination format, where:

- The first-place team from Group A faces the fourth-place team from Group B, and vice versa.
- The second-place team from Group A faces the third-place team from Group B, and vice versa.
- Winners proceed to the semifinals, followed by a final to determine the champion.

Define K_i as the knockout qualification function for team i:

$$K_i = \begin{cases} 1, & \text{if } i \text{ finishes in the top 4 of its group} \\ 0, & \text{otherwise.} \end{cases}$$

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A team qualifies if:

$$R_i \leq 4, \quad \forall i \in G_k, k = 1,2.$$

where R_i is the final league ranking of team i within its group, and G_k represents Group A or Group B. The set of qualified teams K is:

$$K = \{i \mid R_i \leq 4, i \in G_k, k = 1,2\}.$$

The probability of advancing at each stage is dependent on team performance and historical match difficulty, modeled as:

$$P_{win}(i) = \frac{R_{opp}}{R_i + R_{opp}},$$

where R_{opp} represents the opponent's rating. This ensures a structured knockout phase leading to a final league winner.

Choosing Match Locations

League Phase: Home and Away Matches

Each team plays an equal number of home and away matches, ensuring fairness in travel and competitive advantage. The home matches take place at each team's designated home stadium, while away matches are played at the opponent's venue.

Define H_i as the home stadium of team i, and A_{ij} as the distance between the home stadiums of teams i and j. The total travel burden for a team is given by:

$$D_i = \sum_{t=1}^{22} \sum_{j=1, j\neq i}^{10} A_{ij} X_{ijt}.$$

The objective is to ensure that home and away fixtures are evenly distributed, maintaining fairness in travel distance:

$$\sum_{t=1}^{22} \sum_{j=1, j\neq i}^{10} X_{ijt} = 9, \quad \sum_{t=1}^{22} \sum_{j=1, j\neq i}^{10} X_{jit} = 9, \quad \forall i.$$

This ensures that no team has an undue advantage from excessive home games or suffers from excessive travel, thereby optimizing logistical feasibility.

Elimination Round: Choosing a Central Location

The elimination round is played at a neutral central location to ensure fairness and minimize travel fatigue for all qualified teams. The optimal central location is chosen based on multiple factors:

1. **Distance from Qualified Teams:** The central location should minimize total travel distance for all eight qualified teams. Define L_x as a candidate location and $d_{i,x}$ as the distance from team i to location x. The optimal location minimizes:

$$T(x) = \sum_{i=1}^{8} d_{i,x}.$$

2. **Infrastructure and Stadium Quality:** Define S_x as the stadium quality score of location x, normalized between 0 and 1:

$$S(x) = \frac{Q_x}{Q_{\max}},$$

where Q_x represents factors such as seating capacity, training facilities, and hospitality options.

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3. **Fan and Media Coverage:** The location should maximize attendance and media engagement. Define F_x as the expected fan turnout at location x, and M_x as the media coverage potential:
$$C(x) = \alpha F_x + \beta M_x,$$
where α and β are weight coefficients ensuring an optimal trade-off between fan engagement and media reach.
4. **Political and Climate Stability:** Ensuring that the location does not pose risks due to extreme weather or geopolitical concerns. Define P_x as the political stability factor and W_x as the weather suitability score:
$$R(x) = \gamma P_x + \delta W_x,$$
where γ and δ are weight coefficients prioritizing external stability factors.

The overall suitability function for location x is:
$$P(x) = \lambda T(x) + \mu S(x) + \nu C(x) + \xi R(x),$$
where the coefficients λ, μ, ν, ξ sum to 1 and are optimized based on the relative importance of each factor.

The location x^* with the highest $P(x)$ is selected as the elimination round venue:
$$x^* = \arg\max_x P(x).$$
This mathematical framework ensures that the knockout stage is played at a neutral and logistically optimal location, promoting fairness and competitive balance.

Expansion of the Global Soccer League

Integration of Four Additional Teams

To increase the international interest and the level of competition in the Universal Sports League (USL), the number of teams it includes is raised from 20 to 24. Such strategic improvement will guarantee a wider range of competition and the increased diversity of sporting talent representation. The selection criteria of these other teams are set as follows:

- **International Diversity:** Incorporating teams from regions historically underrepresented in the league to foster worldwide engagement and a richer competitive environment.
- **Performance Metrics:** Evaluating each prospective teams's past performance in regional and international competitions, including championships, consistency in top-tier performances, and player development programs.
- **Economic Viability and Audience Reach:** Prioritizing teams with strong financial resources, large fan bases, and high revenue-generation capabilities to ensure long-term sustainability.
- **Operational Readiness:** Evaluation of infrastructure standards, such as stadium capacity, training facilities, and logistical preparedness for international travel and tournament organization.

The four additional teams are located in different continents, reinforcing the league's commitment to improving the inclusiveness and diversity of the league:

- One team from Oceania, enhancing regional representation and global outreach.
- One team from the Middle East, tapping into a rapidly growing sports market with emerging talent.
- One team from Central America, bridging the competition gap between North and South America.
- One team from Eastern Europe, acknowledging the rich sporting heritage and historical significance of the region.

ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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Vol.03 No.04 (2025)

This expansion not only increases the league's competitiveness but also increases its cultural and geographic inclusivity, ultimately driving higher engagement and market growth.

Impact on Seasonal Fixture Design

With the league now accommodating 24 teams, the structural format is redefined into two balanced divisions of 12 teams each. Every team competes against all other teams within its division twice—once at home and once away. Consequently, the number of matches per team increases, given by:

$$T_m = 2 \times (F_d - 1) = 2 \times 11 = 22 \text{ matches per team.}$$

where T_m represents the total number of matches per team, and F_d denotes the number of teams per division.

Mathematical Constraints for Fixture Optimization

To accommodate the increased fixtures while maintaining player well-being and logistical feasibility, the following scheduling constraints are imposed:

- Weekly match frequency is upheld while ensuring fairness:

$$\sum_{w=1}^{30} \sum_{k=1,k\neq j}^{12} Y_{jkw} = 22, \quad \forall j.$$

- Balance in home and away fixtures is maintained:

$$\sum_{w=1}^{30} \sum_{k=1,k\neq j}^{12} Y_{jkw} = 11, \quad \sum_{w=1}^{30} \sum_{k=1,k\neq j}^{12} Y_{kjw} = 11, \quad \forall j.$$

- Player recovery periods are integrated into the schedule:

$$P_w = 1, \quad \forall w \in \{7,14,21,28\}.$$

These constraints optimize fixture balance, prevent excessive fatigue, and ensure a fair allocation of competitive opportunities.

Influence on Championship Determination

With the expanded fixture structure, the accuracy of ranking determination improves due to an increased number of competitive matchups. The probability of a team retaining its rank after g matches is given by:

$$S(g) = 1 - e^{-\mu g}.$$

where μ represents a league-specific ranking stability coefficient. Solving for g when $S(g) \approx 0.95$ yields:

$$g = -\frac{\ln(0.05)}{\mu}.$$

Given $g = 22$, ranking stability significantly improves, minimizing dependencies on tiebreaker mechanisms.

Implications for Travel and Competitive Equilibrium

The inclusion of additional teams necessitates an overhaul of travel logistics. The cumulative travel burden for each team is quantified as:

$$Z_j = \sum_{w=1}^{30} \sum_{k=1,k\neq j}^{12} G_{jk}\, Y_{jkw}.$$

where G_{jk} denotes the geographical distance between teams j and k.

ISSN E: 3006-1466
ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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To maintain competitive parity, an upper bound is enforced on the deviation in cumulative match difficulty across all teams:

$$\max\left(\max_j H_j - \min_j H_j\right) \leq \delta.$$

where H_j represents the difficulty index for team j, and δ is a predefined fairness tolerance threshold. Geographic clustering strategies mitigate excessive travel demands while ensuring fair competitive exposure.

Refinement of Playoff Qualification Standards

Despite the expansion to 24 teams, the postseason knockout format retains an 8-team structure, comprising the top four teams from each division. This sustains the league's existing competitive framework while ensuring logical tournament progression.

The qualification function for team j is defined as:

$$W_j = \begin{cases} 1, & \text{if } j \text{ ranks in the top 4 of its division} \\ 0, & \text{otherwise.} \end{cases}$$

A team qualifies if:

$$O_j \geq O_{min},$$

where O_j represents the final ordinal ranking of team j and O_{min} denotes the qualification threshold (fourth place). The set of qualifying teams, W, is given by:

$$W = \{j \mid O_j \leq 4, j \in D_q, q = 1,2\},$$

where D_q denotes each division. This structured approach ensures a fair and systematic progression to the playoff rounds, preserving both competitive intensity and logistical viability.

Generalizing the Model to Different Team Sports

The mathematical model developed for the Global Sports League (GSL) is designed specifically for football (soccer). However, the core principles of fairness, scheduling, travel optimization, and competitive balance can be adapted to other team sports with varying rules, game durations, and league structures. This section discusses how our model can be generalized to accommodate different sports while maintaining the integrity of the scheduling and competitive framework.

Structural Adaptation for Different Sports

Different team sports have unique characteristics that require modifications to our existing model:

- **Game Duration and Frequency:** Sports like basketball and ice hockey have shorter game durations and allow for more frequent matches per week, requiring adjustments to scheduling density.
- **Scoring Systems and Competitive Balance:** Some sports (e.g., rugby, American football) have different point distribution systems, necessitating modifications in ranking calculations.
- **Team Sizes and Substitutions:** The number of players per team and substitution rules (e.g., rolling substitutions in basketball vs. fixed substitutions in football) influence rest and fatigue modeling.
- **Venue and Infrastructure Requirements:** Sports like cricket require specific pitch conditions, while indoor sports like volleyball and basketball have fewer geographical constraints.

To generalize our model, we redefine key variables based on sport-specific parameters:

$$M = G \times (N - 1) \times F,$$

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ISSN P: 3006-1458
CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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CONTEMPORARY JOURNAL OF SOCIAL SCIENCE REVIEW
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where M is the total number of matches per team, G is the number of groups, N is the number of teams per group, and F is the frequency factor, which depends on the sport's rest and scheduling requirements.

Scheduling Adjustments

To adapt the league schedule for different sports, we introduce:

- **Variable Rest Periods:** Define R_s as the required rest period between games for sport s, ensuring:

$$\sum_{t=1}^{T} (X_{ijt} + X_{jit}) \leq 1, \quad \forall i, j, t \in R_s.$$

- **Dynamic Travel Constraints:** Travel fatigue varies by sport; for example, American football teams require more recovery time after travel. Define T_s as the travel burden factor:

$$D_i^s = \sum_{t=1}^{T} \sum_{j=1, j\neq i}^{N} A_{ij} X_{ijt} T_s.$$

- **Playoff and Knockout Adaptations:** Some sports use best-of-series formats instead of single elimination. Define P_k as the probability of winning a best-of-K series:

$$P_k = \sum_{n=K/2+1}^{K} \binom{K}{n} p^n (1-p)^{K-n},$$

where p is the probability of a single-game win.

Location Selection for Multi-Sport Adaptation

Venue selection varies across sports based on:

- **Field or Court Availability:** Define V_s as a binary indicator for the suitability of a location for sport s.
- **Fan Accessibility and Demand:** Model fan engagement F_x as a function of sport popularity in region x.
- **Broadcast and Media Priorities:** Optimize scheduling to maximize television revenue across different sports.

The optimal location is determined by:

$$P(x) = \alpha T(x) + \beta S(x) + \gamma F(x) + \delta V_s,$$

where weights $\alpha, \beta, \gamma, \delta$ are sport-specific parameters.

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One-page Visual Graphic of the initial 20-team GSL schedule

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