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GENERALIZED EXPONENTIAL ESTIMATORS FOR POPULATION VARIANCE USING RANDOMIZED RESPONSE MODEL

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Abstract

The estimation of population variance for sensitive study variables poses significant challenges due to respondents' reluctance to provide truthful answers. In this study, we develop generalized exponential estimators for estimating the population variance of a sensitive variable by incorporating one and two auxiliary variables within the framework of randomized response models. Using Taylor series linearization and exponential series expansion, we derive the approximate bias and mean square error (MSE) expressions of the proposed estimators. These analytical results allow us to establish optimal conditions under which the new estimators outperform traditional variance estimators available in the literature. The theoretical comparison is supported by inequalities showing the superiority of the proposed methods when certain correlations and design parameters are satisfied. To further validate the performance of our estimators, we conduct a comprehensive simulation study under multiple population structures and varying levels of sensitivity. Additionally, a real data application is presented to demonstrate the practical utility and robustness of the estimators in real-world survey settings. Results from both simulation and empirical analyses confirm that the generalized exponential estimators consistently achieve lower MSE and improved efficiency compared to existing competing models. Overall, this study contributes to the advancement of variance estimation in sensitive surveys by integrating auxiliary information within RRT frameworks and by proposing more efficient generalized exponential estimators for population variance.

Keywords:
Randomized Response Technique (RRT); Sensitive Variables; Population Variance; Exponential Estimators; Auxiliary Information; Bias; Mean Square Error (MSE); Efficiency Comparison; Simulation Study; Real Data Application.

INTRODUCTION

VARIANCE ESTIMATION

Variance estimation of the targeted population is of the major importance in survey sampling. It is assumed to be a key indicator to monitor the process in control. It is considered to be helpful to estimate the overall variability and to reach on valid conclusion by using the variation in the samples. Variance estimation is commonly used to measure the variation in climate change. Different qualities of seeds and soil fertility in agricultural research generally used variance estimation. At the same time, varianc Huang, K.C. (2010).

SENSITIVE STUDY VARIABLE

There are many situations in surveys when our main research variable is sensitive in

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nature and it is difficult to get the true response from the

estimator is also useful to build a valid confidence interval for the estimated parameter of the study.

A general class of quadratic unbiased estimators for the population variance was first proposed by Liu (1974). Later on, Das and Tripathi (1978) gave a class of estimators to estimate population variance. Ratio and regression estimators for population variance were first discussed by Isaki (1983). So far, many researchers have been worked out on variance estimation in different modes of study and the development is continue Balqees,S.(2020).

AUXILIARY INFORMATION

Additional information is usually called the prior information or the auxiliary information which may be obtained from the previous experiments such as past records, surveys, censuses etc. In sampling theory, supporting variable is widely used to increase the efficiency and decrease the error when the study variable is correlated with the supporting variable(s). Nowadays, almost all the researchers inclined to obtain the information of many variables instead of just study variable. The use of the benchmark variable along with the primary variable of interest is very usual to obtain precise estimates for the population parameters such as the mean, median, total, variance, proportion etc. The ratio, product and regression type estimators are the most commonly used estimation methods. Samiuddin and Hanif (2007) presented a variety of procedures to use the auxiliary information in survey sampling.

The use of supplementary information in survey sampling was well described by Tripathi (1970, 1973, 1976). According to him:

i) "It may be used for planning of a survey or at the time of sample design e.g., from information about variable, the population could be stratified etc.

ii) It may be utilized to estimate the population parameters e.g., the ratio, regression, difference, product estimators etc.

iii) It may be utilized for the selection of sample e.g., for calculation of proportional to size probabilities.

iv) It may also be utilized combining any of above two.

respondents directly. Some sensitive study variables are Illegal income, Alcoholism, Homosexual activities, Drug addition, Doping usage, Political views, Spot fixing, Mobbing, Gambling, Abortion, Tax evasion and Sexual and Physical abuse. To estimate the sensitive study variable, randomized response technique is considered to be very useful and efficient Arnab, R. and Dorffner, G. (2006).

RESEARCH QUESTIONS

- Will the proposed estimators beat the existing estimators mathematically and numerically?
- Will the proposed estimators increase the precision and decrease the error of estimation?

STATEMENT OF PROBLEM

From the existing literature, it is observed that a lot of work has been done on the estimation of population mean for sensitive study variable using non-sensitive auxiliary variable(s) but few researchers work is available on the variance estimation using randomized response technique. It is also notice that no contribution has done using exponential estimators along with the randomized response models (RRMs) having one or more than one auxiliary variable for the estimation of population variance having sensitive study variable. In this research, our main concern is to propose some exponential estimators for population variance having sensitive research variable using of one and two benchmark variables under *SRS* design Grover, L.K. (2007).

OBJECTIVE OF THE STUDY

The major objectives of the study are to:

- Suggest the exponential estimators for variance estimation of sensitive study variable in the presence of one or two auxiliary variables using randomized response techniques.
- Derive the expressions of approximate bias and mean square error (*MSE*) of the aforementioned estimators by using Taylor and exponent series.
- Obtained the conditions in which our proposed exponential estimators will perform better than the competing estimators available in literature.
- Assess the performance of proposed estimators using simulation and real data application.

RESEARCHMETHODOLOGY

- In this research we have proposed some exponential estimators for sensitive study variable using auxiliary variables for population variance using well-known Taylor and exponential series."
- A widespread simulation and real-data illustration are conducted using R-software to judge the implementation of the proposed exponential estimators over the competing estimators

LITERATURE REVIEW

Yousaf (2020) suggested a class of ln-type mean estimators for the estimation of sensitive study variable using the information of variance of the auxiliary variable under simple and stratified sampling. The proposed classes of estimators were compared with the ordinary mean and Sousa *et al.* (2010 and 2014) estimators empirically and theoretically. The results showed better performance of the proposed class of estimators than the existing estimators for both real and artificial populations.

Anwar (2020) suggested the generalized estimators for sensitive study variable using the non-sensitive auxiliary attribute in simple and stratified random sampling. The expressions of bias and MSE of the proposed estimator were also derived and compared with the classical estimators (ordinary mean, classical ratio, regression, exponential ratio and exponential product estimators). The results of simulation and empirical study showed that the generalized estimators are more efficient than the classical estimators under both simple and stratified random sampling design.

Rana (2021) proposed the generalized estimators for confidential study variable using auxiliary attribute in two-phase simple and stratified sampling. The expressions of bias and MSE of the proposed estimator were also derived and compared with the mean, ratio, and exponential estimators. The results of numerical study revealed that the generalized estimators performed well than the existing estimators under two-phase simple and stratified sampling designs.

Material and Method

In this chapter, we have discussed randomized response techniques and randomized response models along with their basic properties (means and variances). Some exponential estimators are also mentioned for non-sensitive study and the auxiliary variables for population mean and

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variance. The expressions of bias and MSE are also given with their respective estimators.

RANDOMIZED RESPONSE MODELS

The RRT is an advance technique that aims to get the almost accurate answers to a sensitive question that respondents might be hesitant to answer truthfully, such as use of drugs, sexual practice, illegal earning, incidence of acts of domestic violence, or in collusion with terrorists or extremists nationally or internationally and many others are included in surveys of human populations. Warner (1965) was the first to suggest an ingenious method of counteracting fears in response to sensitive questions and called it RRT. He developed an interviewing procedure designed to reduce or eliminate the bias. Later, several other authors have suggested various alternative randomized response strategies.

Some well-known models are discussed below:

Additive Model

The additive model first given by Pollock and Beck (1976) where the related response is obtained by adding the scrambling variable in the main survey variable

$$Z \quad Y \quad S, \qquad (1)$$

Where$Z$ isthe reported response, Y be the variable of interest and S is the scrambling variable. Further, it is assumed $S \sim N$ (0, 1%* S_x).

The mean and variance of sensitive study variable is:

$$E(Z) = E(Y),$$

And

$$Var(Z) = Var(Y) + Var(S),$$

Multiplicative Model

Eichhron and Hayre (1983) first gave the idea of multiplicative model to get the scrambled response. The model is defined as

$$Z \quad Y * T, \qquad (2)$$

where T is the scrambling variable and Z and Y are as usual sensitive study and the non-sensitive auxiliary variables respectively. It is assumed that $E(T) = 1$

The mean and variance of sensitive study variable is

$$E(Z) = E(Y),$$

And

$$Var(Z) = Var(Y).Var(T),$$

General Linear Model

Diana and Perri (2011) proposed a more general linear model using two scrambling variables. The model is defined by adding and multiplying two different scrambling variables simultaneously with the confidential study Saha, A. (2008).

The mean and variance of general linear model are respectively given by

$$E(Z) = E(Y),$$

And

$$S_z^2 = S_y^2 + S_s^2\left(\mu_y^2 + S_y^2\right) + S_T^2.$$

SOME EXISTING EXPONENTIAL ESTIMATORS

In the past, many authors have proposed exponential estimators for the estimation of population parameters having non-sensitive study variable. In this section, we have mentioned some existing estimators for population mean and variance of sensitive study variable in *SRS*design. The usual unbiased variance estimator along with the variance is defined as

$$t_y = s_z^2 - S_s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}\left(z_i - \bar{z}\right)^2 - S_s^2,$$

and

$$\mathrm{var}\left(t_y\right) = \theta\lambda_{40}^*.$$

Classical ratio estimator for the estimation of population variance of confidential variable is defined as

The expressions of approximate Bias and *MSE* of t_r are

$$Bias\left(t_r\right) = \theta S_z^2\left[\lambda_{04}^* - \lambda_{22}^*\right],$$

and

$$MSE\left(t_r\right) = \theta S_z^4\left[\lambda_{40}^* + \lambda_{04}^* - 2\lambda_{22}^*\right].$$

The ratio estimator having two auxiliary variables for estimating the mean of population variance of sensitive study variable is defined as

$$t_r^* = \left(s_z^{*2} - S_s^{*2}\right)\left(\frac{S_x^2}{s_x^2}\right)\left(\frac{S_u^2}{s_u^2}\right).$$

The expressions of approximate Bias and *MSE* of t_r are

$$Bias\left(t_r^*\right) \approx \theta S_y^2\left[\lambda_{040}^* + \lambda_{004}^*\right] - \theta S_z^2\left[\lambda_{022}^* + \lambda_{202}^*\right],$$

and

$$MSE\left(t_r^*\right)=\theta S_z^4\lambda^* + \theta S_y^4\left[\lambda^*_{0400}+\lambda^*_{04}+2\lambda^*_{022}-2\Re_{zy220}\left(\lambda^*_{202}+\lambda^*_{400}\right)\right].$$

Omer (2020) suggested exponential type ratio estimator for estimating the population variance of a confidential variable using single benchmark variable is defined as

$$t_{e11}=s_z^2 exp\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right).$$

The equations of bias and *MSE* of t_{er} are respectively given by

$$Bias\left(t_{e11}\right)=\frac{1}{8}\theta S_z^2\left[3\lambda^*_{04}-4\lambda^*_{22}\right],$$

and

$$MSE\left(t_{e11}\right)=\theta S_z^4\left[\lambda^*_{40}+\frac{1}{4}\lambda^*_{04}-\lambda^*_{22}\right].$$

The exponential ratio estimator for estimating the population variance of a sensitive study variable using two auxiliary variables is defined as

$$t^*_{e11}=s_z^2 exp\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}+\frac{S_u^2-s_u^2}{S_u^2+s_u^2}\right).$$

The expressions of bias and *MSE* of t_{er} are respectively given by

$$Bias\left(t^*_{e11}\right)=\frac{1}{4}\theta S_y^2\left[\lambda^*_{040}+\lambda^*_{004}+\lambda^*_{022}\right]-\frac{1}{2}\theta S_z^2\left[\lambda^*_{220}+\lambda^*_{202}\right],$$

and

$$MSE\left(t^*_{e11}\right)=\theta S_z^4\lambda^*_{400}+\theta\frac{1}{4}S_y^4\left[\lambda^*_{040}+\lambda^*_{004}+2\lambda^*_{022}-4\Re_{zy}\left(\lambda^*_{220}+\lambda^*_{202}\right)\right].$$

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Results and discussion

In this chapter, we have proposed an exponential estimator for sensitive study variable using supplementary information operating *RRM*. Mathematical terminologies of bias and *MSE* of the proposed exponential estimator are derived using Taylor and exponential series. The expression of minimum *MSE* of the proposed exponential estimator is also derived using the optimization of the constant. Few special cases are also obtained as the family of the proposed estimator using different values of the scalar constants Gupta, S., et al (2020).

PROPOSED GENERALIZED EXPONENTIAL ESTIMATOR

In this section, we have proposed a generalized exponential estimator for the estimation of population variance of confidential variable using benchmark auxiliary information in SRS design. The estimator is defined as

$$t_{eji} = \left(s_z^2 - S_s^2\right) exp\left(F \frac{\left(\alpha S_x^{2b^{-1}} + \beta\right) - \left(\alpha s_x^{2b^{-1}} + \beta\right)}{\left(\alpha S_x^{2b^{-1}} + \beta\right) + (f-1)\left(\alpha s_x^{2b^{-1}} + \beta\right)} \right), \quad (4.3.1)$$

where b ($b>0$) and $F(F)$ are two real constants and supposed to be known. Further, 0 and be the scalar quantities which can take distinctive groupings of real number and known parameters of benchmark variable to modify the form of proposed exponential estimators. The optimized constant *"f"* is used to catch the minimum *MSE* of the proposed generalized exponential estimator

MATHEMATICAL COMPARISONS

In this section, we mathematically compared the proposed exponential estimator with the existing estimators.

- The proposed exponential estimator will perform better than the unbiased variance estimator if

$MSE_{min} \quad t_{eji} \qquad var \quad t_y \quad .$

NUMERICAL STUDY

In this section, we have conducted a numerical study to check the performance of the proposed exponential estimator over the existing estimators considered in this research. We have conducted the simulation study for the population generated by normal distribution with different mean and covariance matrix. Further, we also used the real data-set of information and communication technologies population, which was initially used by Sousa *et al.* (2010) to defend the mathematical properties of the proposed exponential estimator. The efficiency of the proposed estimators is evaluated on the basis of absolute bias (AB), MSE and percentage relative efficiency (PRE). The formulas of absolute bias,*MSE*and *PRE*are defined a

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Simulation Study

In this sub-section, we have conducted a simulation study to evaluate the performance of the proposed generalized exponential estimators using a bivariate normal population with means and variance covariance matrix to represent the distribution of Y and X. The characteristics of populations are:

able 4.1

Theoretical and Empirical *AB*, *MSE* and *PRE* of all the Estimators for Artificial Population

Empirical Theoretical							
N	*n*	***Estimators***	***AB***	***MSE***	***AB***	***MSE***	***PRE***
5000	500	t_y	0.00985	0.12753	0.00875	0.12299	100.0000
		t_r	0.00322	0.06863	0.00212	0.06408	191.9257
		t_{eji}	**0.01102**	**0.06182**	**0.01002**	**0.06046**	**198.4746**
		t_{e1i}	0.00905	0.06852	0.00795	0.06398	192.2334
		t_{e2i}	0.01007	0.09064	0.00897	0.08610	142.8484
		t_{e3i}	0.00331	0.06842	0.00221	0.06388	192.5448
		t_{e4i}	0.01012	0.10724	0.00902	0.10270	119.7616
		t_{e5i}	0.00888	0.09403	0.00777	0.08949	137.4362
		t_{e6i}	0.00980	0.10932	0.00869	0.10478	117.3839
		t_{e7i}	0.00444	0.07226	0.00334	0.06771	181.6355
		t_{e8i}	0.00993	0.11806	0.00883	0.11352	108.3458
		t_{e9i}	0.01325	0.06827	0.01215	0.06372	193.0089
		t_{e10i}	0.01155	0.06847	0.01045	0.06392	192.4026
		t_{e11i}	0.01661	0.24124	0.01550	0.23670	51.96098
		t_{e12i}	0.01070	0.09062	0.00960	0.08608	142.8790

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Empirical Theoretical							
N	*n*	***Estimators***	***AB***	***MSE***	***AB***	***MSE***	***PRE***
5000	1000	t_y	0.00773	0.06665	0.00662	0.06211	100
		t_r	0.00859	0.03704	0.00749	0.03249	191.1432
		t_{eji}	**0.00523**	**0.03132**	**0.00443**	**0.03054**	**199.9556**
		t_{e1i}	0.00689	0.03630	0.00579	0.03176	195.5868
		t_{e2i}	0.00700	0.04759	0.00589	0.04304	144.2919
		t_{e3i}	0.00857	0.03698	0.00747	0.03244	191.4652
		t_{e4i}	0.00728	0.05615	0.00618	0.05160	120.3577
		t_{e5i}	0.00767	0.04933	0.00657	0.04479	138.6761
		t_{e6i}	0.00748	0.05722	0.00638	0.05268	117.9033
		t_{e7i}	0.00936	0.03817	0.00825	0.03362	184.7116
		t_{e8i}	0.00755	0.06174	0.00645	0.05720	108.581
		t_{e9i}	0.00354	0.03695	0.00244	0.03241	191.6226
		t_{e10i}	0.00563	0.03628	0.00453	0.03174	195.6956
		t_{e11i}	0.00283	0.13058	0.00172	0.12604	49.27626
		t_{e12i}	0.00668	0.04758	0.00558	0.04304	144.3134
5000	2000	t_y	0.00187	0.02663	0.00077	0.02209	100
		t_r	0.00331	0.01500	0.00221	0.01046	211.2243
		t_{eji}	**0.00035**	**0.009821**	**0.00025**	**0.009344**	**215.6432**
		t_{e1i}	0.00213	0.01522	0.00102	0.01067	206.9603
		t_{e2i}	0.00188	0.01953	0.00078	0.01498	147.4338
		t_{e3i}	0.00331	0.01499	0.00221	0.01045	211.3767
		t_{e4i}	0.00185	0.02273	0.00074	0.01819	121.4611
		t_{e5i}	0.00210	0.02018	0.00100	0.01563	141.2976
		t_{e6i}	0.00190	0.02313	0.00080	0.01859	118.8541
		t_{e7i}	0.00297	0.01593	0.00187	0.01139	193.9629
		t_{e8i}	0.00187	0.02481	0.00076	0.02027	108.9846
		t_{e9i}	0.00145	0.01498	0.00035	0.01044	211.6174
		t_{e10i}	0.00166	0.01522	0.00056	0.01067	206.9686
		t_{e11i}	0.00118	0.04795	0.00008	0.04341	50.89057
		t_{e12i}	0.00177	0.01953	0.00066	0.01498	147.4325

From the results summarized in Table-4.1, we observed that the proposed class of exponential estimator performed better than the non- exponential estimators for the estimation of population variance of sensitive study variable in the presence of non-sensitive auxiliary variable except t_{e11i}. Moreover, the precision of the proposed class of exponential estimator increases by the increase of the sample size. The estimators, t_{e3i}, t_{e9i}, and t_{e10i}performed well than the existing estimators and the other members of the proposed class of exponential estimator. Consequently, we may say that the proposed class of exponential estimator is superior then the other existing estimators for different sample sizes.

Real Data Application

In this section, we have considered a real population based on the survey of information and communication technologies (*ICT*) usage in initiatives in 2009 seat in Portugal (Smilhily and Storm 2010). Three different sample sizes (n = 1000, 2000 and 3000) are considered to assess the performance of the estimators considered in this research. The theoretical and empirical*AB*,and *MSE* of all the estimators are given in Table4.2.

Table 4.2
Theoretical and Empirical *AB*, *MSE* and *PRE*
of all Estimators for Real Population

Empirical Theoretical							
N	*n*	***Estimators***	***AB***	***MSE***	***AB***	***MSE***	***PRE***
		t_y	4.81	216264.25	4.78	216263.50	100.00
		t_r	5.40	33453.75	5.36	33453.00	646.47
		t_{eji}	**5.23**	**33412.32**	**5.28**	**33413.43**	**648.98**
5336	500	t_{e1i}	6.51	72468.55	6.47	72467.80	298.43
		t_{e2i}	6.01	131258.35	5.98	131257.60	164.76
		t_{e3i}	5.40	33437.97	5.37	33437.22	646.77
		t_{e4i}	5.50	170483.05	5.46	170482.30	126.85

			Empirical		***Theoretical***		
N	*n*	***Estimators***	***AB***	***MSE***	***AB***	***MSE***	***PRE***
		t_{e5i}	5.22	139413.45	5.18	139412.70	155.12
		t_{e6i}	5.26	175248.85	5.22	175248.10	123.40
		t_{e7i}	3.68	83266.13	3.65	83265.38	259.73
		t_{e8i}	5.09	195108.45	5.06	195107.70	110.84
		t_{e9i}	11.02	33491.43	10.98	33490.68	645.74
		t_{e10i}	7.91	72459.55	7.88	72458.80	298.46
		t_{e11i}	17.21	269106.85	17.18	269106.10	80.36
		t_{e12i}	6.36	131252.55	6.33	131251.80	164.77
5336	1000	t_y	23.09	1322309.75	23.06	1322309.00	100.00
		t_r	19.46	208133.35	19.43	208132.60	635.32
		t_{eji}	**18.98**	**207042.15**	**19.01**	**207043.40**	**651.43**
		t_{e1i}	12.41	430084.15	12.38	430083.40	307.45
		t_{e2i}	15.55	791948.35	15.51	791947.60	166.97
		t_{e3i}	19.23	207283.15	19.19	207282.40	637.93
		t_{e4i}	18.77	1036007.75	18.73	1036007.00	127.64
		t_{e5i}	20.53	842902.85	20.50	842902.10	156.88
		t_{e6i}	20.29	1065886.75	20.25	1065886.00	124.06
		t_{e7i}	30.17	496436.95	30.13	496436.20	266.36
		t_{e8i}	21.31	1189891.75	21.27	1189891.00	111.13
		t_{e9i}	15.95	209336.85	15.92	209336.10	631.67
		t_{e10i}	3.58	429521.75	3.55	429521.00	307.86
		t_{e11i}	54.40	1748382.75	54.37	1748382.00	75.63
		t_{e12i}	13.34	791643.45	13.30	791642.70	167.03
1000	2000	t_y	82.39	1416382.75	82.35	1416382.00	100.00
		t_r	61.58	209175.25	61.55	209174.50	671.45
		t_{eji}	**60.95**	**207931.05**	**60.98**	**208001.40**	**682.58**
		t_{e1i}	81.52	450344.85	81.49	450344.10	314.51
		t_{e2i}	84.52	842355.95	84.49	842355.20	168.15

Empirical Theoretical							
N	*n*	***Estimators***	***AB***	***MSE***	***AB***	***MSE***	***PRE***
		t_{e3i}	61.91	208099.05	61.88	208098.30	680.63
		t_{e4i}	83.95	1106564.75	83.91	1106564.00	128.00
		t_{e5i}	79.65	897368.65	79.62	897367.90	157.84
		t_{e6i}	82.66	1138858.75	82.63	1138858.00	124.37
		t_{e7i}	63.81	521842.05	63.78	521841.30	271.42
		t_{e8i}	82.94	1273090.75	82.90	1273090.00	111.26
		t_{e9i}	99.59	210015.35	99.55	210014.60	674.42
		t_{e10i}	91.00	449901.85	90.96	449901.10	314.82
		t_{e11i}	116.41	1867118.75	116.38	1867118.00	75.86
		t_{e12i}	86.69	842097.45	86.66	842096.70	168.20

As expected, the numerical findings gain from the simulation study support the outcome obtained in the real data illustration. The results presented in Table-4.2 showed that the proposed class of estimator is more efficient than the existing estimators at different sample sizes. The estimator, t_{e3i}, executed well among all the existing and other members of the proposed class of exponential estimators. Thus, from the results of real data study, we conclude that the proposed class of estimator is efficient for estimating the population variance of a sensitive study variable using benchmark variable in *SRS* design

except t_{e11i}. On the basis of numerical results, the generalized exponential estimator found to be more superior than all the estimators used in this research at different sample sizes.

CONCLUSION

There are many situations in real-life surveys when the study variable is sensitive or confidential in nature. The questions based on such sensitive variables can raise the refusal to the answer or fabricated answers given intentionally. The estimates obtained from a direct survey on sensitive questions would be subject to high response bias. In this research, we have proposed an exponential estimator for sensitive study variable in the presence of auxiliary variable. The exponential estimators can produce families of estimators using different choices (real numbers or the parameters of the auxiliary variable) of scalar constants. We have also proposed exponential estimator using two auxiliary variables using randomized response technique. The properties of the proposed estimators are derived up to the second-order approximation using well- known Taylor and exponential series. The equations of minimum *MSE* of the proposed exponential estimators are obtained by differentiating the equations of *MSE* with respect to the optimized parameters. Various families and sub-families of the proposed estimators are obtained using different choices of generalized constants. The optimum conditions are also obtained in which the proposed estimators are compared over the classical variance and ratio estimators.

A broad numerical study is conducted using real and artificial populations generated through bivariate normal distribution. From the numerical findings, it is concluded that the exponential estimators perform outstanding for both real and artificial populations on different sample sizes for the estimation of scrambled responses under certain conditions. The exponential ratio type estimators perform better for the population having positive correlation with the supporting variable whereas the exponential product estimators perform well for negatively correlated population. Accordingly, on the basis of numerical study, we may conclude that the proposed exponential estimators are superior to the classical variance estimator at different levels of correlation for the estimation of finite population variance under *SRS* design.

In this study, we have considered the exponential-type estimators for estimating the population variance of a sensitive study variable using randomized responses having single scrambling variable. In future, we will consider generalized estimator based on exponential and non-exponential functions and different randomized response models with two scrambling variables in the presence of multi-auxiliary variables.

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